ON THE UNCERTAINTY OF LINEAR ENERGY TRANSFER SPECTRA MEASURED WITH TRACK-ETCHED DETECTORS IN THE SPACE

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THE GOAL

• determination of measurement uncertainty for particular LET spectra

• track-etched detectors as spectrometers of LET in space

• material PADC TD1
LET SPECTRA

• two-stage process of LET spectra determination:

1. **Calibration**: determination of the calibration curve \( L = f(V) \), where \( V = f(\text{track parameters}) \) with particles of known LET

2. **Measurements**: determination of LET spectrum
LET SPECTRA: UNCERTAINTIES

• three categories of independent uncertainties:

  1. uncertainty associated with randomness of particle detection \( u_1 \)
  2. uncertainty of the calibration curve \( u_2 \)
  3. uncertainty of the detector response \( u_3 \)

• resulting uncertainty

\[
\begin{align*}
  u &= \sqrt{u_1^2 + u_2^2 + u_3^2}
\end{align*}
\]
UNCERTAINTIES OF PARTICLE DETECTION

- number of tracks in channel - different due to random nature of particle detection

- Poisson distribution with channel uncertainty \( u_{1i} = \sqrt{N_i} \)

- usually the only considered uncertainty
UNCERTAINTIES OF THE CALIBRATION CURVE

- confidence interval of calibration curve

$u_{2i} = \Delta L \left| \frac{dn_L}{dL} \right| u(L)$

• shifts of peaks in spectrum (uncertainty associated with spectrum channel)
UNCERTAINTIES OF THE DETECTOR RESPONSE

- the LET specific species are associated with random channel number
- Gaussian distribution

\[ u_3 \]

**note the log x scale (the same std deviation of all Gauss)**
METHODS: factors affecting uncertainties

**Measured quantities uncertainty**
1. measuring with microscope
2. non-uniform etching conditions
3. detector thickness differences
4. differences in operator’s view
5. separation of peak of primary particles (calibration)
6. separation of “the biggest” fission fragments

**Calculated quantities uncertainty**
7. equation uncertainty
8. reference calibration LET values
9. angle dependence
10. calibration curve model
11. ...
23 samples were irradiated with $^{252}$Cf, selection of tracks for analysis:

- perpendicularly impinging fission fragments: $b/a > 0.9$
- heavy fragments ($A \sim 143$) against light ($A \sim 99$): operator-specific routine
Kernel density function of the parameter B corresponding to heavy fragments (dashed line) and normal distribution (full line) with parameters

- $B = 14.86 \mu m$
- $u(B) = 0.28 \mu m$
- relative standard uncertainty: 1.9 %
• V - LET
• HIMAC-BIO
• He, C, Ne, Si, Ar, Fe, Kr
• 31 calibration points
• uncertainty of a, b: normal distribution with parameters a, u(a), b, u(b), u(a,b)
Selection of tracks for analysis:

- $b_{\text{min}} < b < b_{\text{max}}$
- $0.9 < b/a < 1$

The example with measured tracks of 400 MeV/u C
CALIBRATION

- etch ratio calculation
  \[ v = \sin^{-1}\left[ \frac{B}{2a \sqrt{1 - \frac{b^2}{B^2}}} \right] \]

- sensitivity coefficients
  \[ c_1 = \frac{\partial v}{\partial a}, \quad c_2 = \frac{\partial v}{\partial b}, \quad c_3 = \frac{\partial v}{\partial B} \]

- combined variance
  \[ u^2_c(V) = c_1^2 u^2(a) + c_2^2 u^2(b) + c_3^2 u^2(B) + 2c_1 c_2 u(a, b) \]
### CALIBRATION

<table>
<thead>
<tr>
<th>LET keV.µm⁻¹</th>
<th>V</th>
<th>uᵥ(V)</th>
<th>uᵥ_rel(V) %</th>
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<thead>
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<th>LET keV.µm⁻¹</th>
<th>V</th>
<th>uᵥ(V)</th>
<th>uᵥ_rel(V) %</th>
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</table>
• \( V = f(L) \)
• \( L = f(V) \) constructed as inverse function

\[
u(L) = u(V) \left| \frac{dV}{dL} \right|^{-1}
\]

• should respect physical characteristics of TED: detection threshold, saturation
• different models lead to different confidence intervals
CALIBRATION: DIFFERENT MODELS

- third degree polynomial
- piece-wise linear
DETECTOR RESPONSE

- smearing of „true signals“
- assumption: minor uncertainties

TEST

- convolution with Gaussian distribution – parameters dependent on LET (fit of the calibration data)
- real LET spectrum from ISS: equidistant eight bins
\( u_3 = 0 \)

\[ u = \sqrt{u_1^2 + u_2^2} \]
\[ u_1(n_\lambda) \]
\[ u_2(n_\lambda) \]
\[ u_c(n_\lambda) = \sqrt{u_1^2 + u_2^2} \]
<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$n\lambda$</th>
<th>LET (keV.µm$^{-1}$)</th>
<th>$\text{relu}_1$</th>
<th>$\text{relu}_2$</th>
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CONCLUSIONS

• calibration model has to be considered very carefully, with respect to physical properties of detectors (which are?)

• uncertainty associated with detector response is difficult to estimate

• uncertainty associated with angle dependence