Dosimetry with thin silicon detectors

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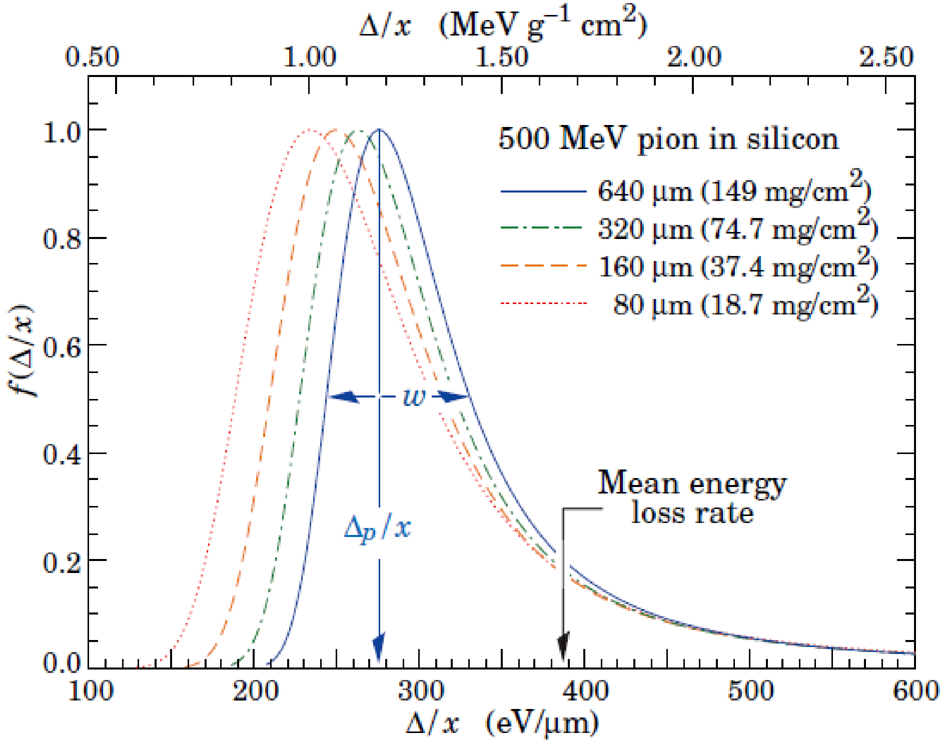
Main Topics

- Calibration
- Vavilov/Landau vs. Bethe
- Converting Si dose to tissue dose
  - Modeling
  - Insights from RAD (with caveats)

LET: Measured vs. Ideal

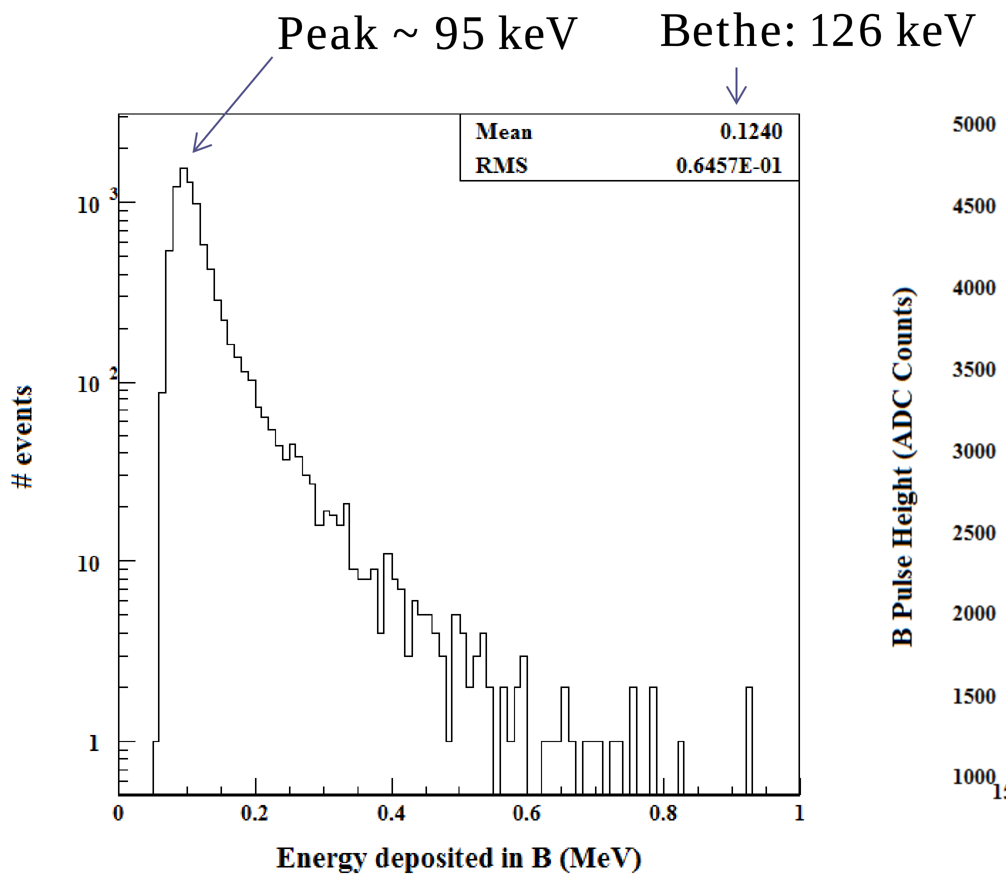
- Bethe formula is deterministic, gives $\langle dE/dx \rangle$.
- Full Vavilov probability distribution has same average as Bethe, $\langle LET \rangle_{Vavilov} = \langle dE/dx \rangle_{Bethe} = LET_{\infty}$.
- If distribution truncated, $\langle LET \rangle_{Vavilov} < LET_{\infty}$.
- Truncation of Vavilov distribution is a function of detector thickness and/or cuts made in data analysis.

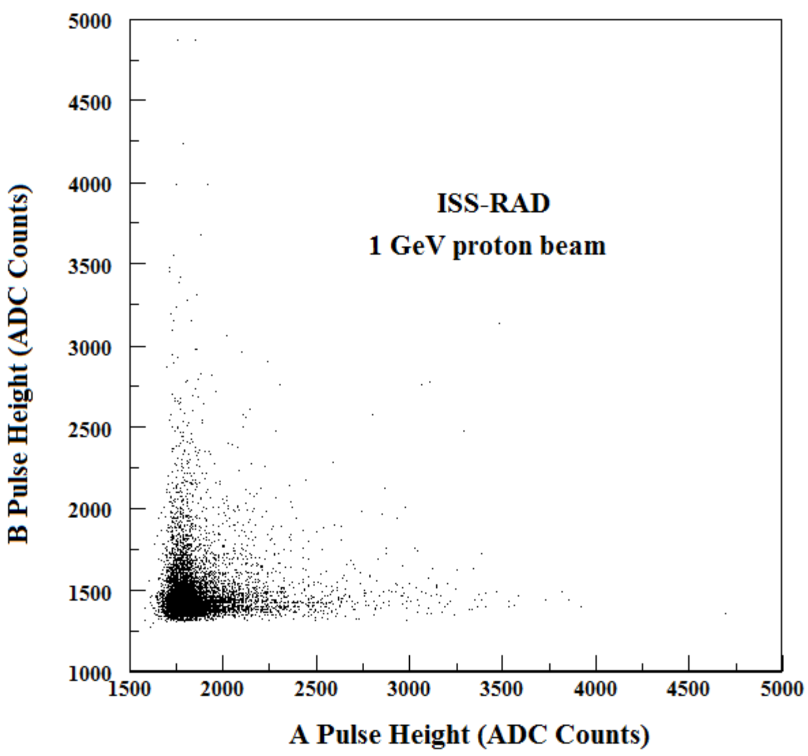
Calibration of Thin Silicon Detectors



- If peaks from minimum-ionizing charge-1 particles are used for calibration, associate peak with “most probable” energy loss rather than the mean.
  - True both for flight data and accelerator data.

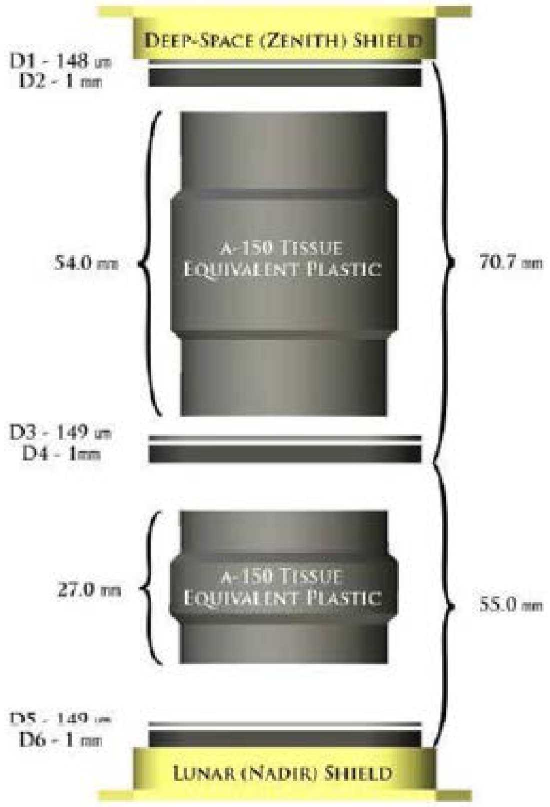
Data: 1 GeV protons in 300 μm Si





Compare Ions @ 1 GeV/nuc in CRaTER

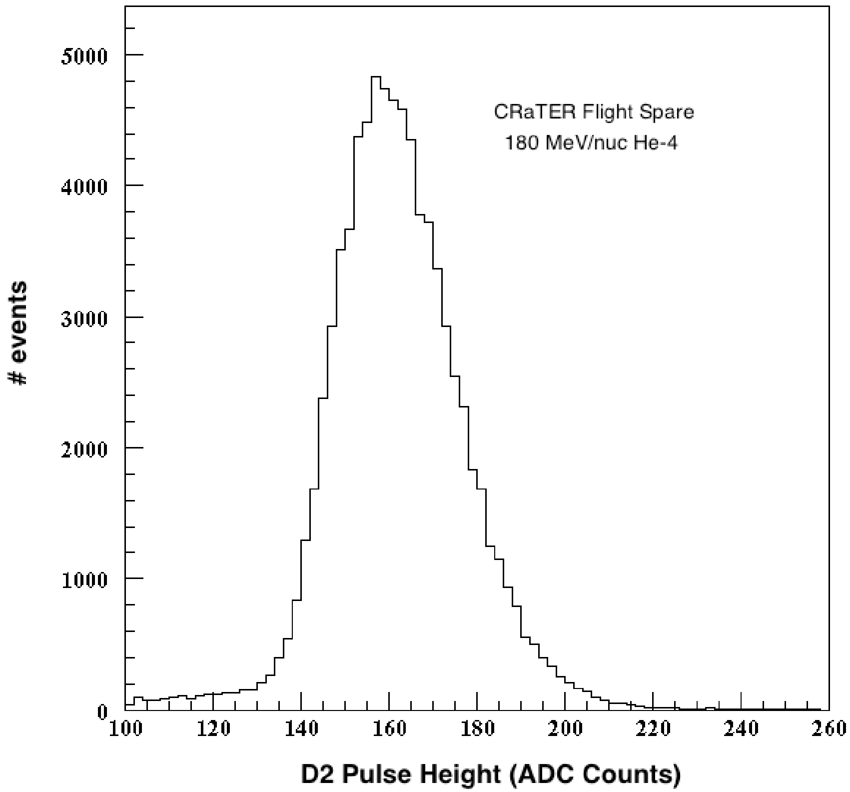
Ion	150 µm peak/average	1 mm peak/average
H	0.68	0.79
He	0.75	0.86
C	0.87	0.98
Mg	0.95	1.05
Fe	1.03	1.14



- Ratio always larger for thicker detector, increases as energy deposition increases.
- As peak → mean, Vavilov → Gaussian.
- Peak/average values > 1 don't make sense.

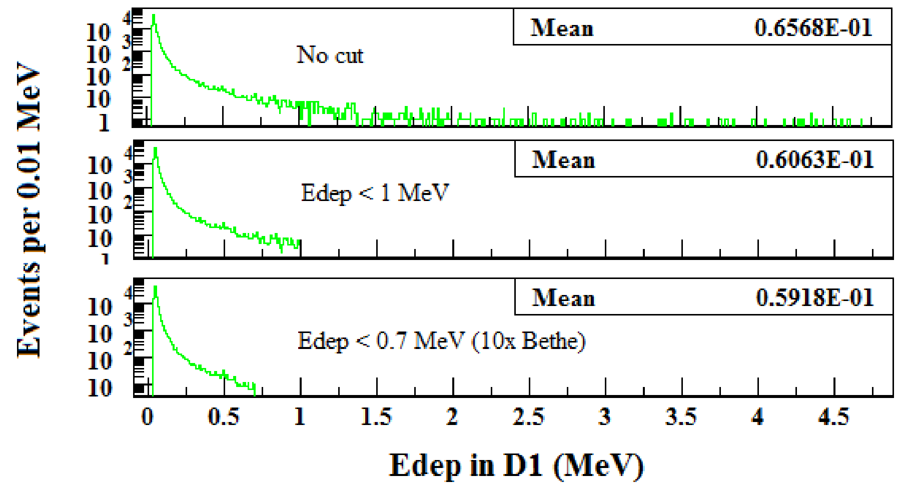
Calibration with Heavy Ions

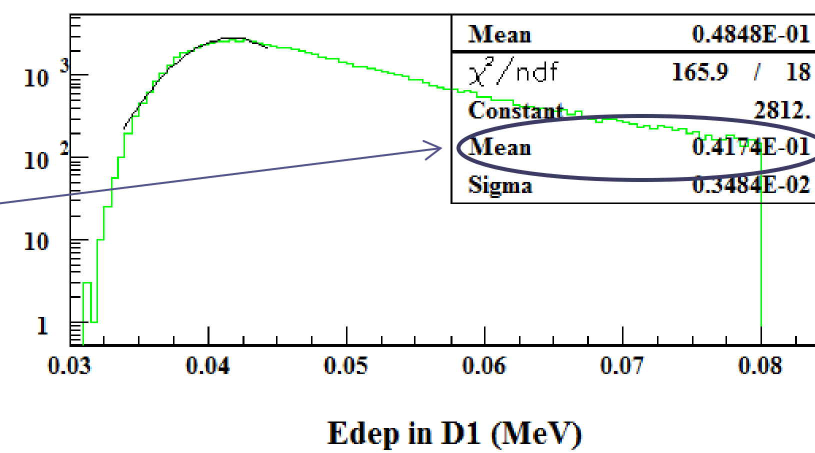
- Problem: Vavilov formalism fails for Z > 1.
- See this in 180 MeV/nuc ^{4}He data from HIMAC in 2012 taken with CRaTER flight spare:
  - Measured ΔE distributions in 1 mm detectors are ~ Gaussian, predicted distributions are skewed.
- Probably best to use Bethe.



Vavilov Avg. vs. Bethe Formula

- Results for simulated 1 GeV protons on 148 μm of Si (D1 in CRaTER).
- Expected <ΔE> from Bethe = 62 keV.
  - My code uses GEANT3 Landau distribution, gives ~ 66 keV (100k events).
- Mean shifts when high end tail is truncated → Implications for dosimetry.
- As check of code, look for peak.
  - Find ~ 41.7 keV, Bichsel formula gives 42 keV.



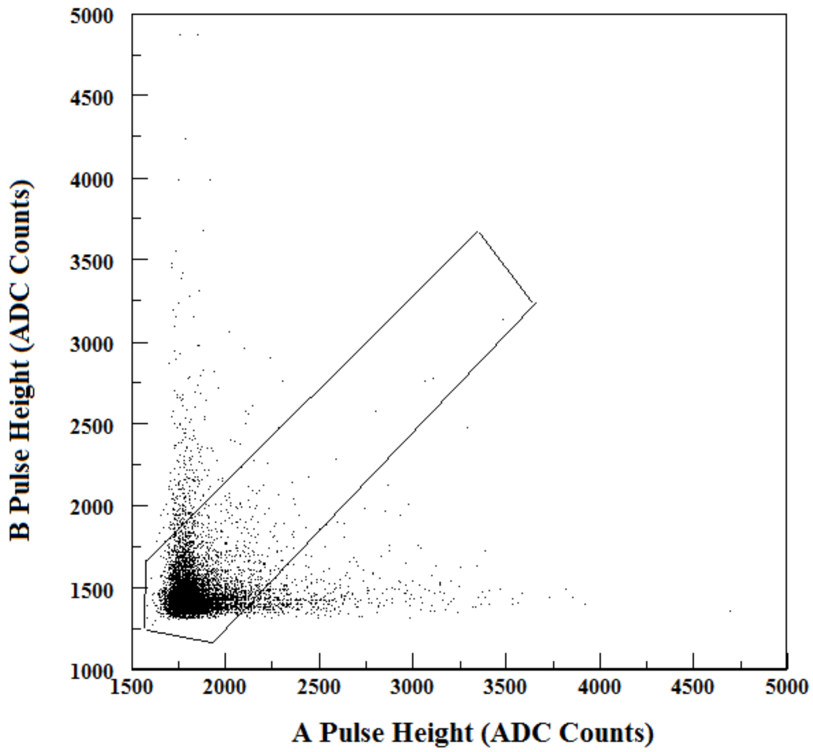


Underlying Physics

- High-end tail of energy deposition arises from rare collisions with large energy transfers to single e^-.
- $T_{max} \approx 2m_e\beta^2\gamma^2$ for ions.
- For example, @ 1 GeV/nuc, $(\beta\gamma) \sim 1$ so $T_{max} \sim 1$ MeV.
  - Range of 1 MeV e^- in Si = 2.3 mm.
  - 4 MeV ΔE in 148 μm of Si requires ~4 maximum-energy transfer collisions.
- Statistics of small numbers → Poisson distribution.
  - Vavilov resembles Poisson.
  - As LET increases and/or detector gets thicker, "rare" collisions become less rare, Poisson → normal.

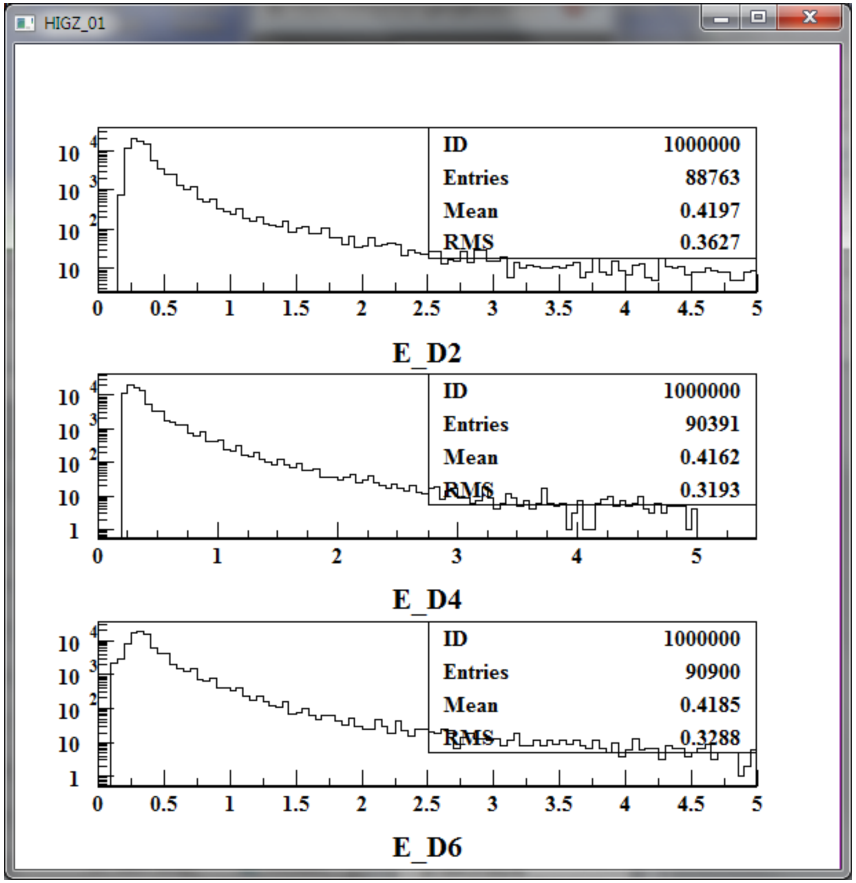
Truncation of Vavilov Distribution

1. Electrons escape & carry off energy – unavoidable.
2. In some analyses, require mutually consistent hits in detector pairs.
   - E.g., in MSL-RAD onboard LET spectrum, two detectors are checked and they must be mutually consistent to within a factor of 2.



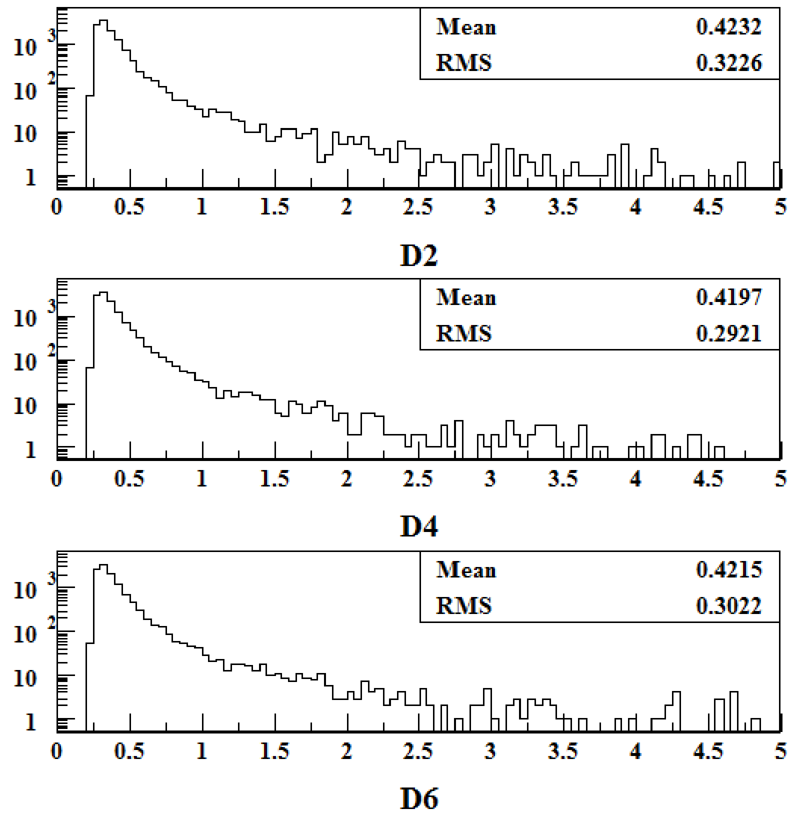
Vavilov Distributions in CRaTER Data

- Pick small, random GCR sample (Jan. 1-10, 2012).
- For D2 plot, require D4 and D6 to have 0.1 – 0.5 MeV ΔE, so we have high-energy charge-1 particles ~ parallel to detector axis.
- Similar for other plots.
- Get ~ same distribution in all 3, mean is ~ 0.42 MeV and RMS is ~ 0.3 MeV.



Compare to Simulated Protons

- 25k simulated GCR events.
- Distributions all drop off above 5 MeV in simulation.
  - Slightly truncated in code.
- Averages close to GCR data, RMS's ~ 10% smaller.
- Vavilov distributions are seen in flight data & they can be simulated reasonably well.



Silicon to Water LET Conversion

Slide by M. Golightly

LET_{Si} → LET_{H_2O} Conversion

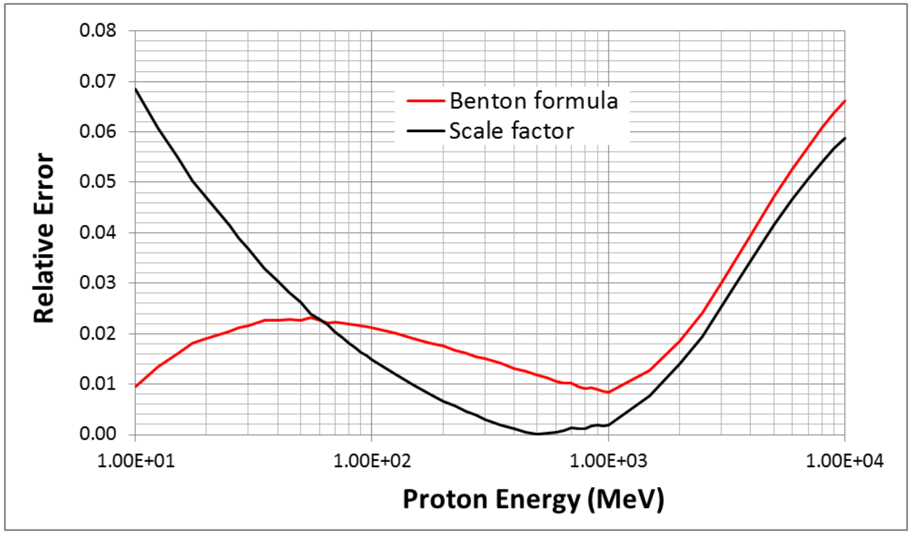
Benton, E.R., E.V. Benton, and A.L. Frank. "Conversion between different forms of LET." *Rad Meas*, **45(8)**, (2010) pp 957-9.

$$\log(LET_\infty H_2O) = -0.2902 + 1.025\log(LET_\infty Si)$$

- Based on Henke and Benton's range/energy relations of ions in H_2O and Si (Henke and Benton, 1967; Benton and Henke, 1969)
  - Z = 1 to 26, E = 0.8-2000 MeV/amu
- Ratio $LET_\infty H_2O : LET_\infty Si$ varies
  - 30% for E = 0.8-2000 MeV/amu
  - 5% for E = 50-2000 MeV/amu
- Functional relationship of $LET_\infty H_2O$ to $LET_\infty Si$ obtained from least squares fit to data

Fit vs. Simple Scaling for LET_∞

- Factor of ~ 1.23 is often used to multiply silicon dose to give tissue dose.
- For GCR it may be more accurate to use the simple scale factor.
- Benton formula has smaller errors below 50 MeV, but for E > 80 MeV, the scale factor is better (avg. err = 1.4% vs 2.3%).
  - Most GCRs have E > 80 MeV.
- May be best to use a hybrid for RAD & CRaTER data.



$$\text{Relative Error} = |LET_{true} - LET_{calc}| \,/\, LET_{true}$$

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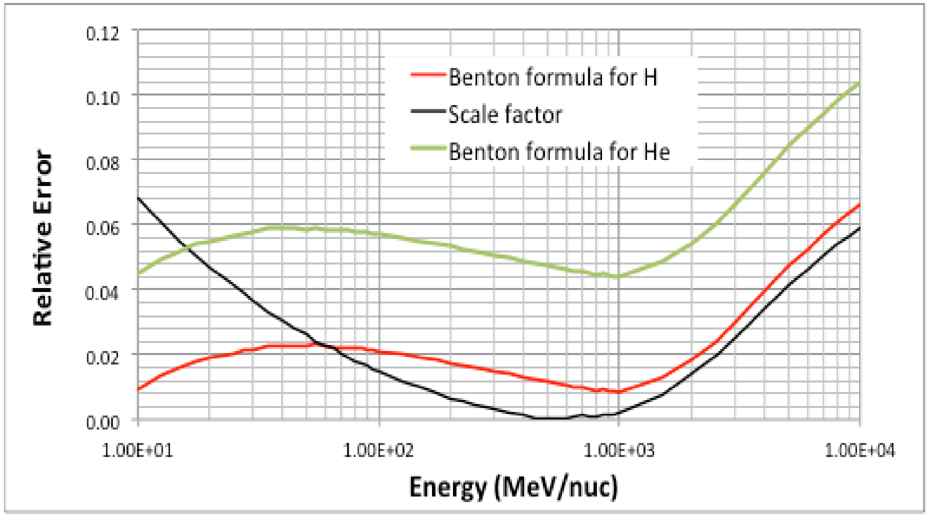


Relative Error = $|LET_{true} - LET_{calc}| / LET_{true}$

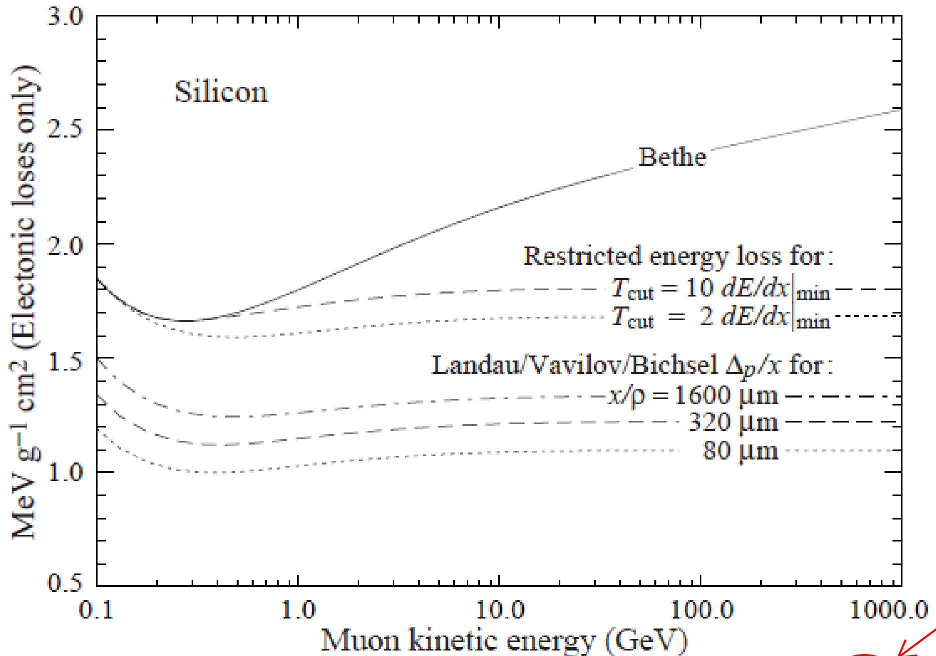
Recap

- Bethe formula gives correct average ΔE's.
- Actual distributions are Vavilov for high-energy, low-charge ions.
  - Vavilov distribution has same mean as Bethe provided full distribution is captured.
- For dosimetry, escape of e^- or truncation by other means → measured LET < LET_∞.
- Si to tissue factor of 1.23 assumes LET_∞ in Si.

How to Correct for LET < LET_∞?

- LET < LET_∞ → factor needs to be larger…but how much larger?
- In earlier simulations, I used restricted dE/dx to compensate for e^- escape.
  - Off a bit.

Restricted Energy Loss Theory & Implementation



$$-\left.\frac{dE}{dx}\right|_{T<T_{\text{cut}}} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2 T_{\text{cut}}}{I^2} - \frac{\beta^2}{2}\left(1+\frac{T_{\text{cut}}}{T_{\text{max}}}\right) - \frac{\delta}{2}\right].$$

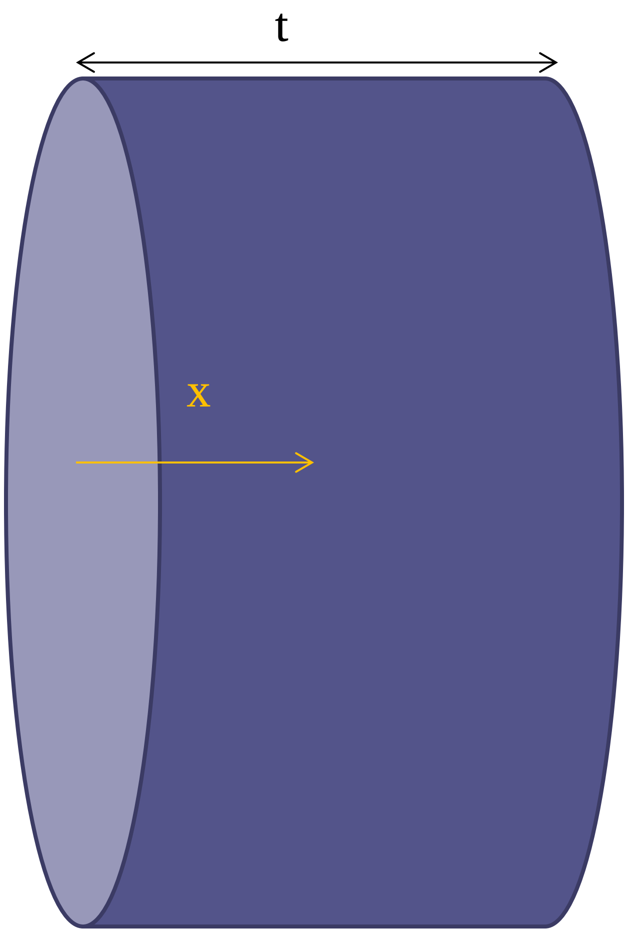
Replaces T_{max}

$T_{max} \approx 2m_e\beta^2\gamma^2$ for ions.

Get Bethe formula back if $T_{\text{cut}} = T_{\text{max}}$

What to Use for T_{cut}?

- In simulation, particles are followed along paths through the detector in 1 μm steps.
- In each step, calculate energy of an electron (E) with range R equal to the remaining depth of the detector E(R(t – x)).
- This value is used for T_{cut}.
- **Overestimates escape**.
  - Treats delta-electrons as if they are forward-produced at exactly 0°.
  - Ignores multiple scattering.

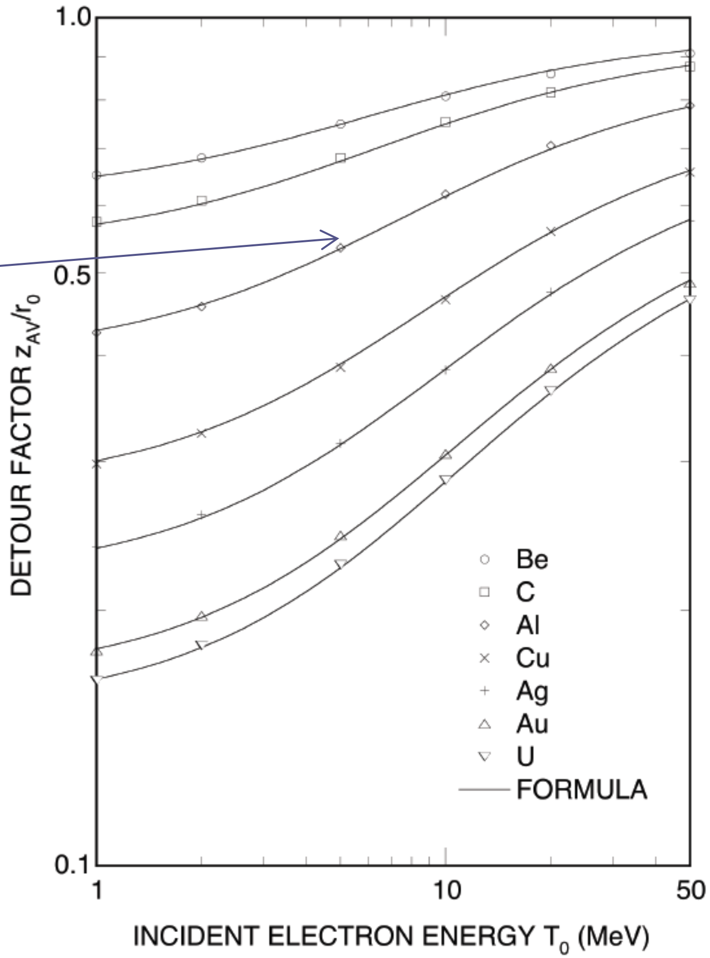


Upshot

- RAD cruise paper: Dose conversion factor of 1.45 (±0.2) based on restricted dE/dx with low T_{cut}.
- Making T_{cut} more realistic changes this result.
  - Include approximation of "detour factor" for electrons.
  - Detour factor accounts for multiple scattering of e^- in the detector (tends to keep them in the detector).

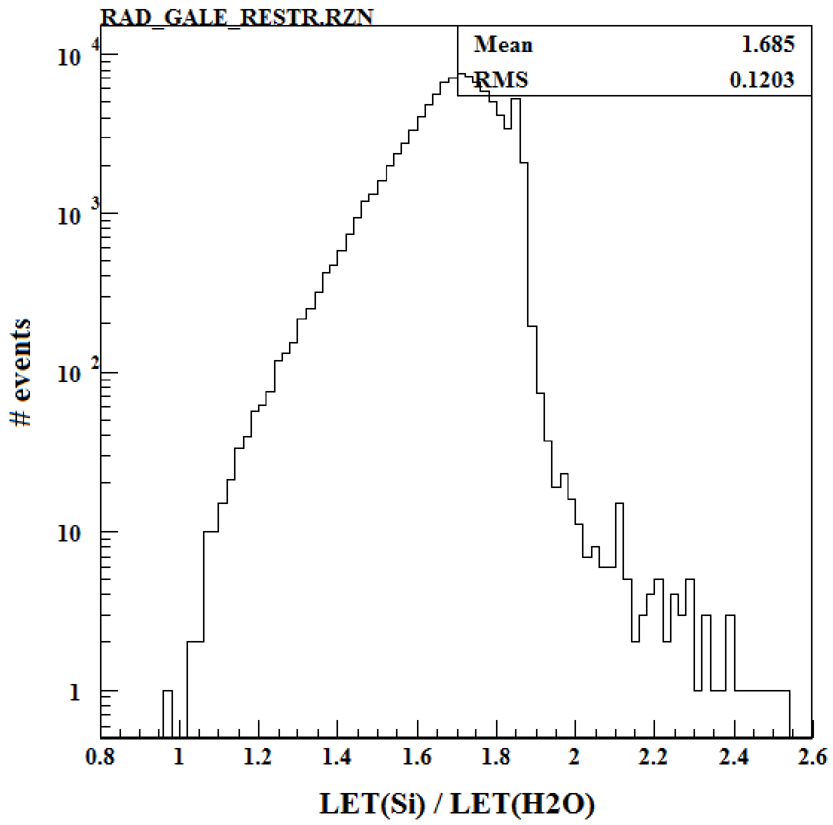
Tabata and Andreo (1998)

- Formula for detour factor in the 1-50 MeV range for electrons in elements.
- Aluminum close to Si.
- Detour factor is projected range divided by CSDA range – projected paths are ~ factor of 2 shorter than nominal.
    - Put factor of 2 into code used to calculate T_{cut} in restricted energy loss formalism.



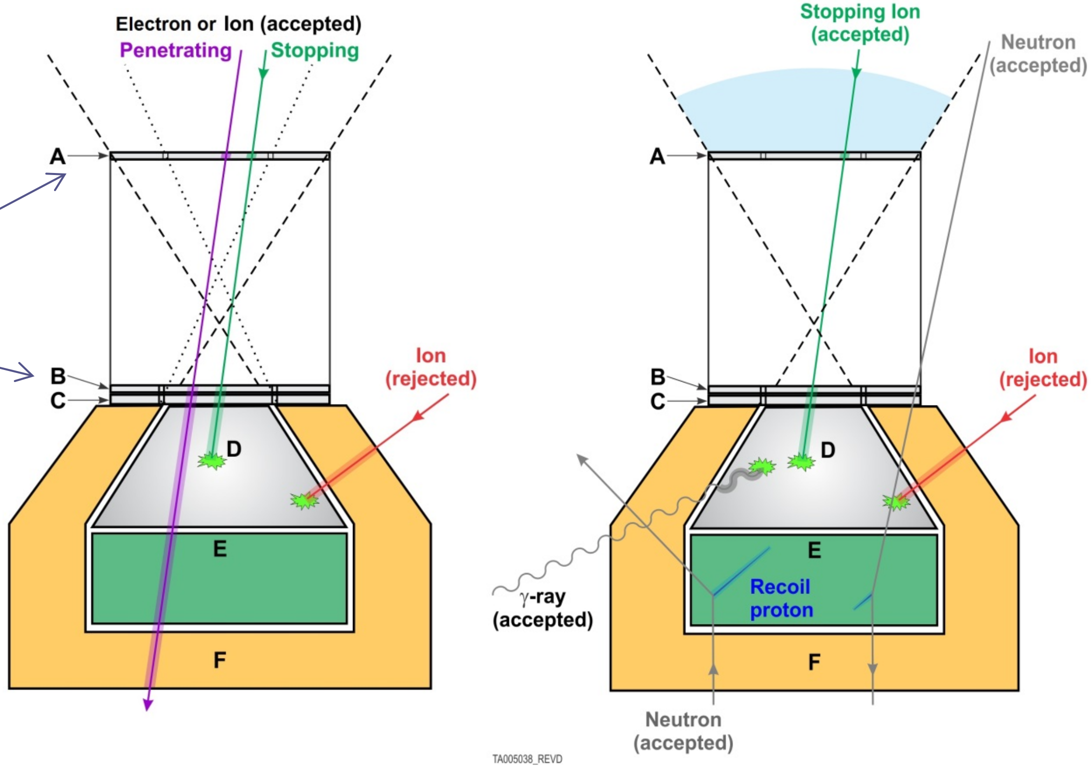
New RAD Simulation Result

- Simulate GCR (B-O spectrum) with 20 g cm^{-2} CO_2 above RAD.
- Compare LET in B detector to LET in water at entrance window above RAD.
- Dose conversion factor becomes 2.33/Mean = 1.38.
  - Distribution asymmetric, RMS ~ 7% of mean.
- Check sensitivity of result to detour factor:
  - Value of 3 instead of 2 gives conversion factor of 1.36.



Insight from RAD Data

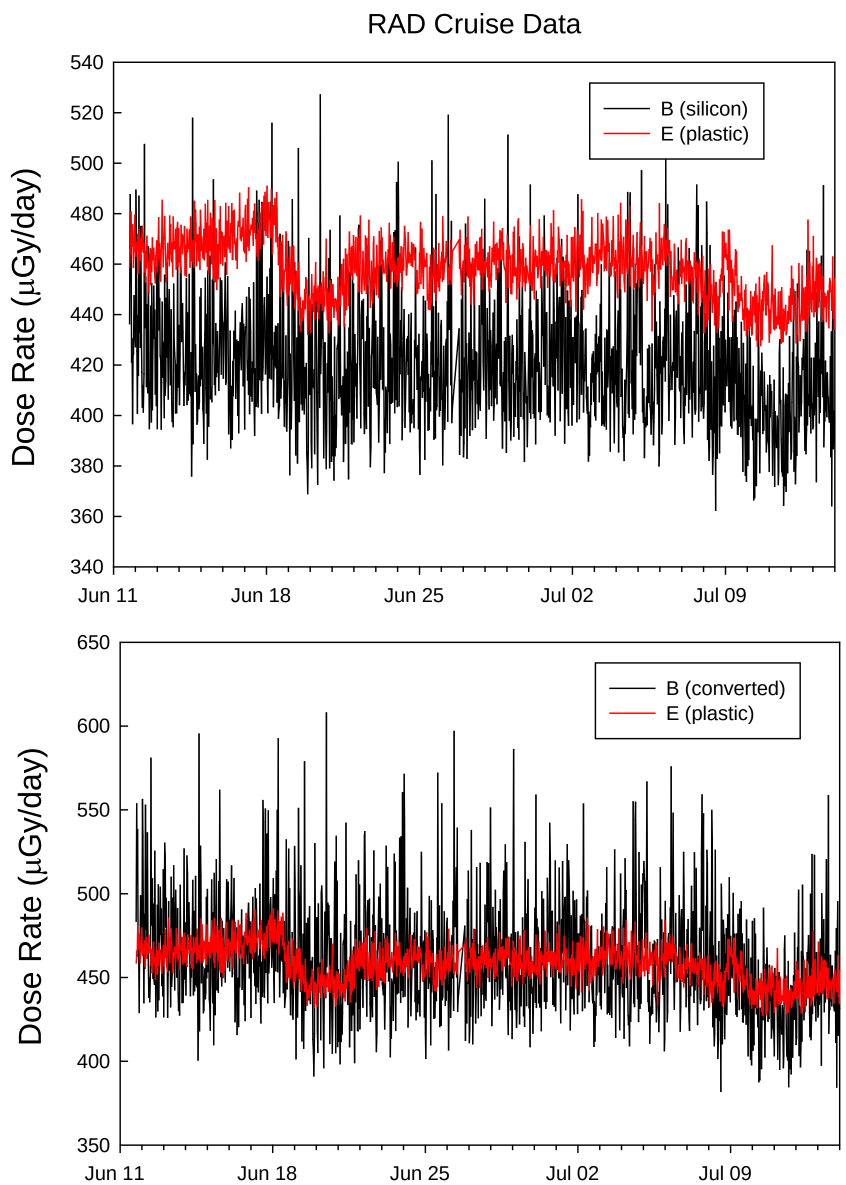
A, B, C are 300 μm Si



- RAD B (silicon) and E (plastic) detectors are used for dosimetry.
  - All hits above threshold contribute, regardless of hits in other detectors.
- Differences:
  - E misses some charged particle dose (stoppers in D or F).
  - E is more sensitive to neutrons, B is more sensitive to gammas.
  - Mass of E is ~ 300× mass of B.

RAD Cruise Data

- RAD was moderately shielded in cruise, ~ 16 g cm^{-2} on average.
- E dose was ~98% due to charged particles during solar quiet time with ~ 0 RTG background.
- E calibration is dominant uncertainty (quenching).
- Additional caveats:
  - B has RTG background.
    - Measured @ Cape; subtracted from flight data in lower plot.
  - E is plastic, not water (2% effect).
- Scale factor of 1.37 makes B and E average dose rates equal.



Conclusions, Next Steps

- Si → water factor of 1.23 is too small because measured LET in Si < LET_∞.
- Si → water factor of 1.45 for 300 μm Si is too big (overestimated electron escape).
- Revised calculation with detour factor=2 → 1.38.
- RAD data suggest 1.37, with several caveats.
- For CRaTER, factor is larger for thin detectors (150 μm), smaller for thick (1 mm).
- Still working on simulation, RAD E calibration.