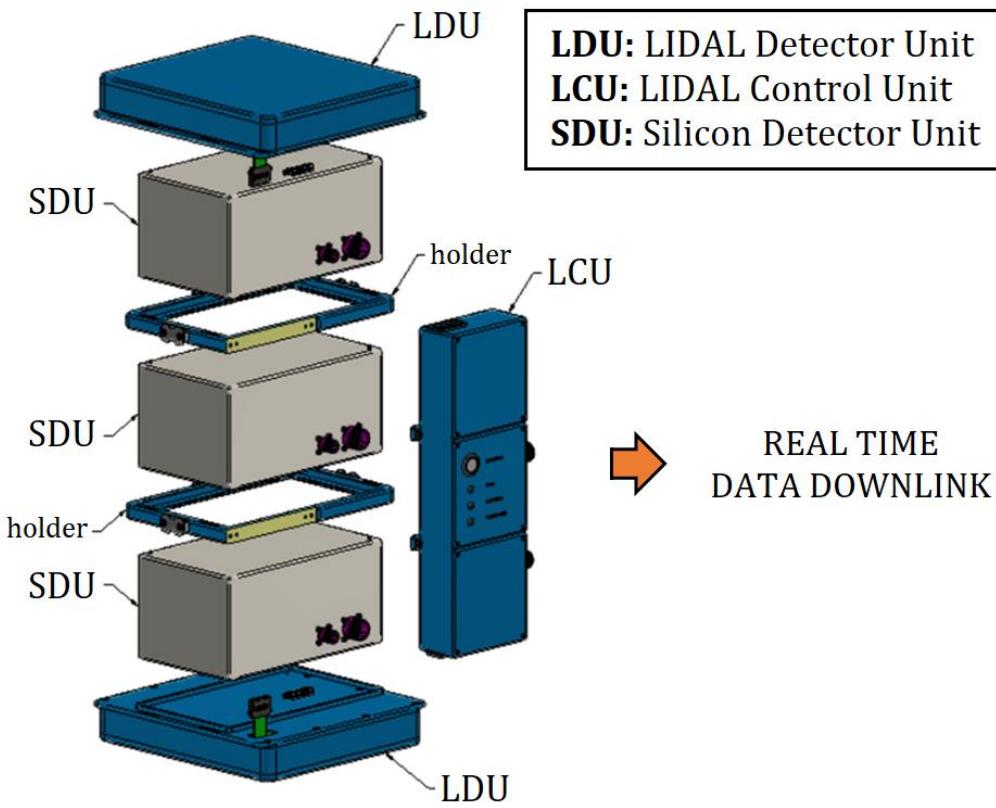


LIDAL: exploiting the Bethe Bloch equation for single particle energy identification

Romoli G.* on behalf of the LIDAL collaboration

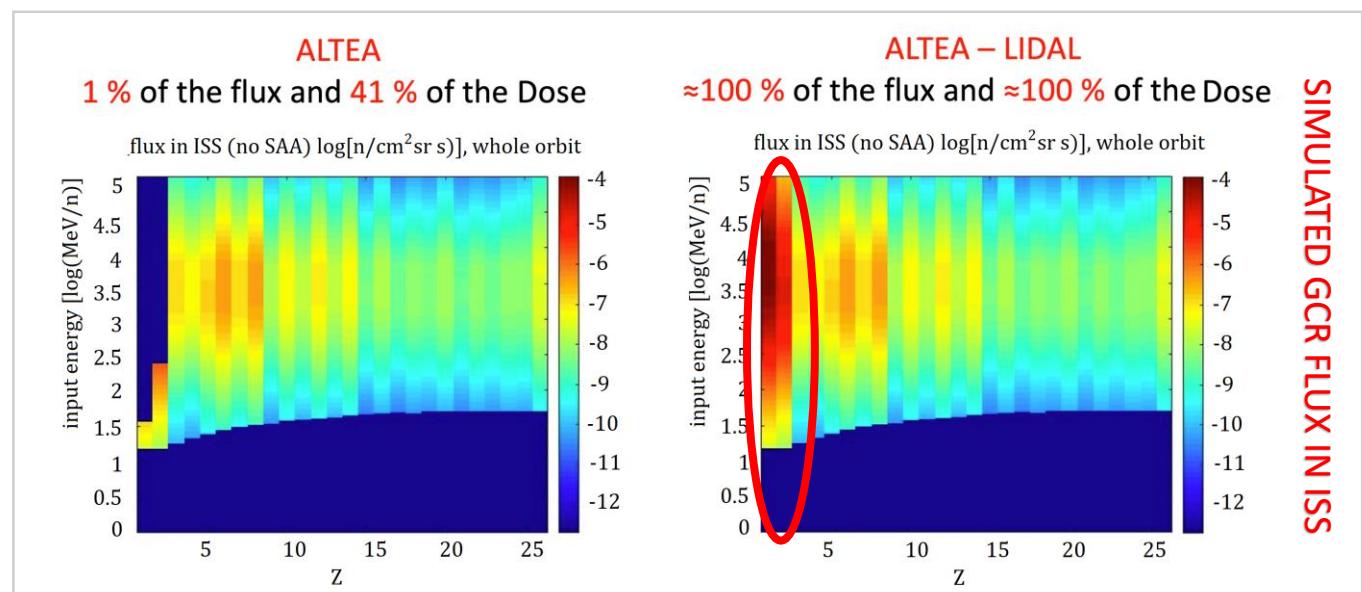
* giulia.romoli@roma2.infn.it

The LIDAL detector



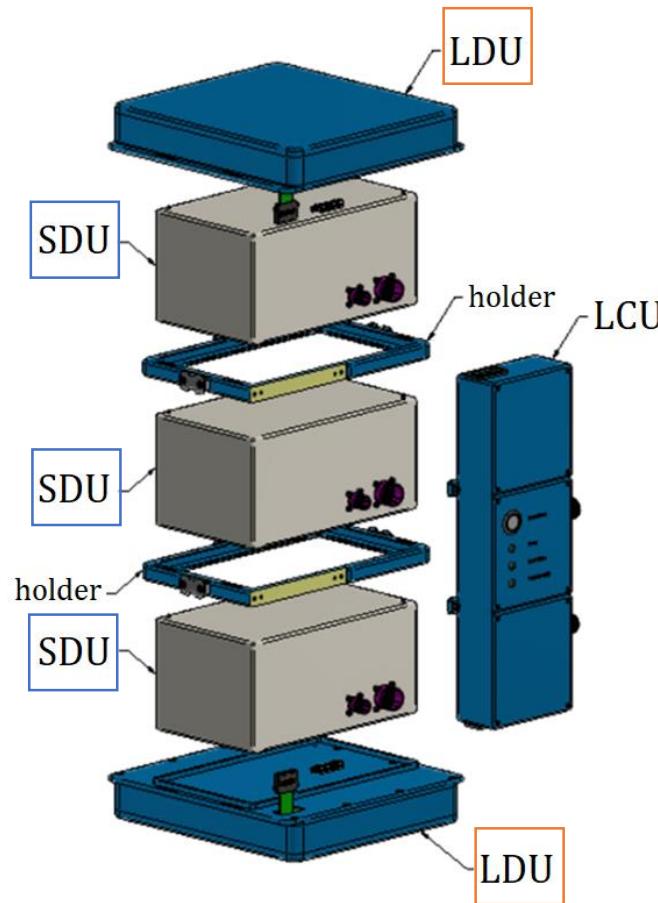
LIDAL (Light Ion Detector for ALTEA):

- enhanced telescope capabilities
(18 silicon planes + 2 scintillator planes)
- first measurement of TOF ($\rightarrow z^2/\beta^2$) in space
- expand acceptance window of ALTEA for light nuclei

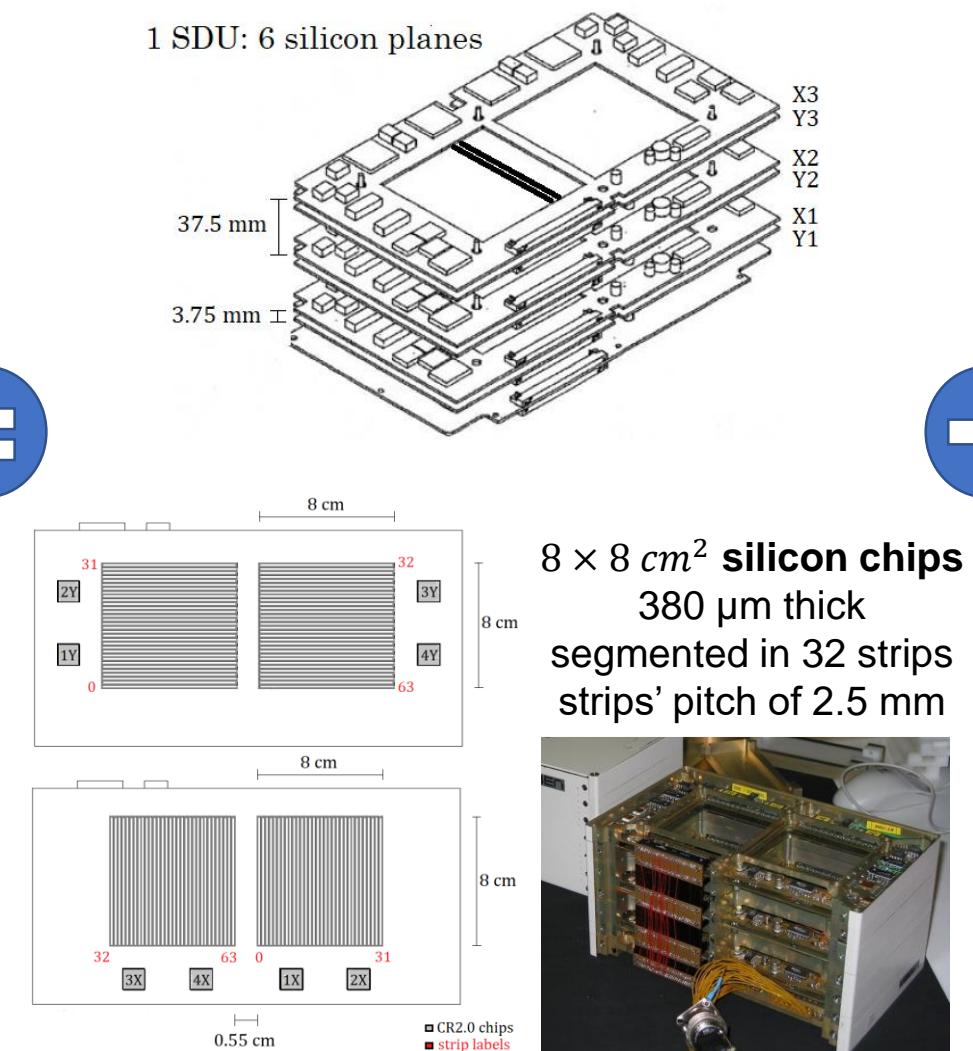


The LIDAL detector

LID AL

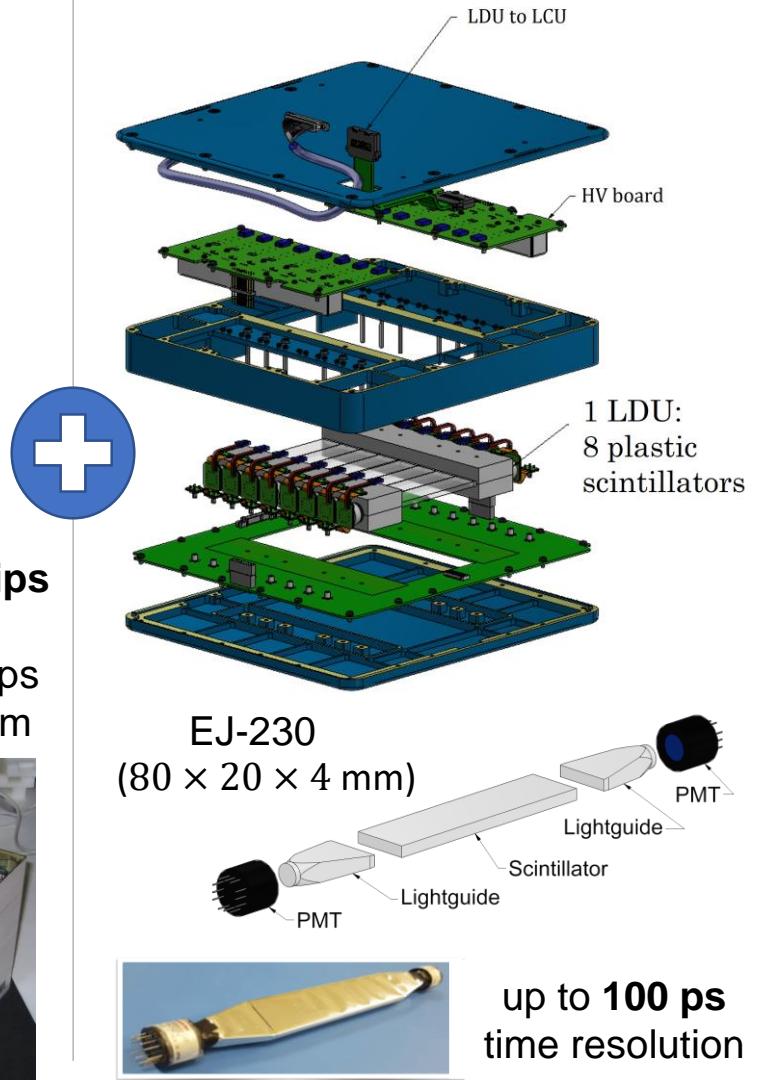


THE ALTEA SUBSYSTEM (3 SDUs)

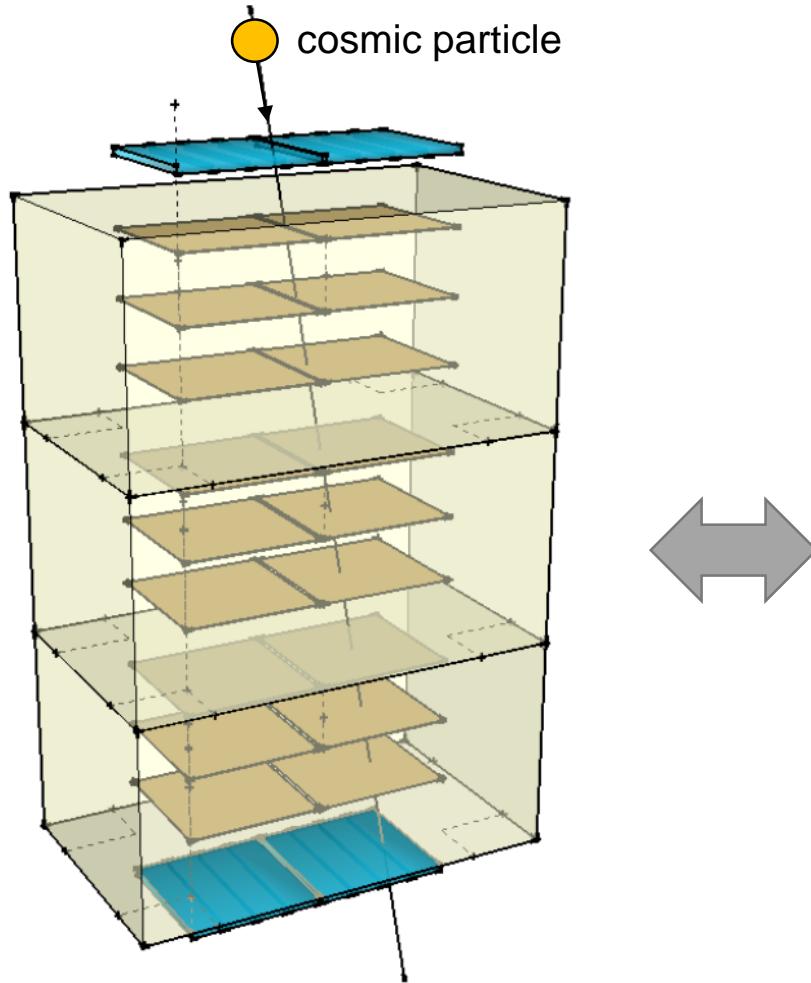


GF: 15.32 $\text{cm}^2 \text{ sr}$ (bi-directional)

THE LID SUBSYSTEM (2 LDUs)



Summary



INCIDENT PARTICLE PROPERTIES

ATOMIC NUMBER Z
INITIAL KINETIC ENERGY

BETHE-BLOCH

LAYERS PROPERTIES

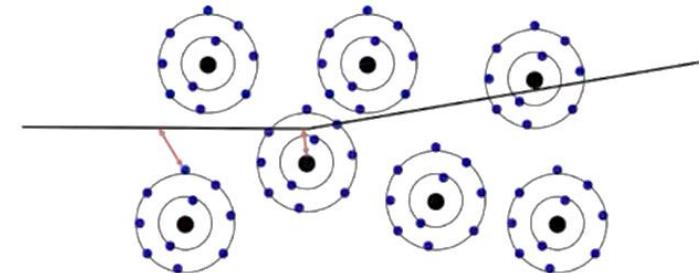
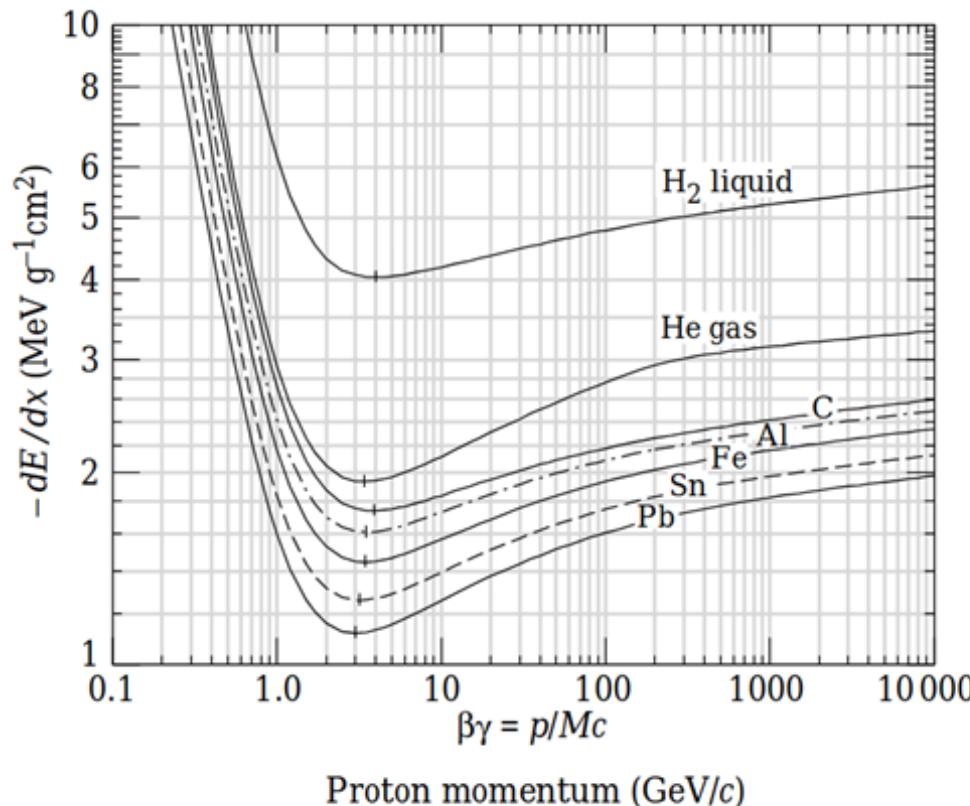
CHEMICAL COMPOSITION
THICKNESS

ENERGY PROFILE $E(x)$

VELOCITY PROFILE $v(x)$

TIME OF FLIGHT PROFILE $T(x)$

The Bethe-Bloch equation

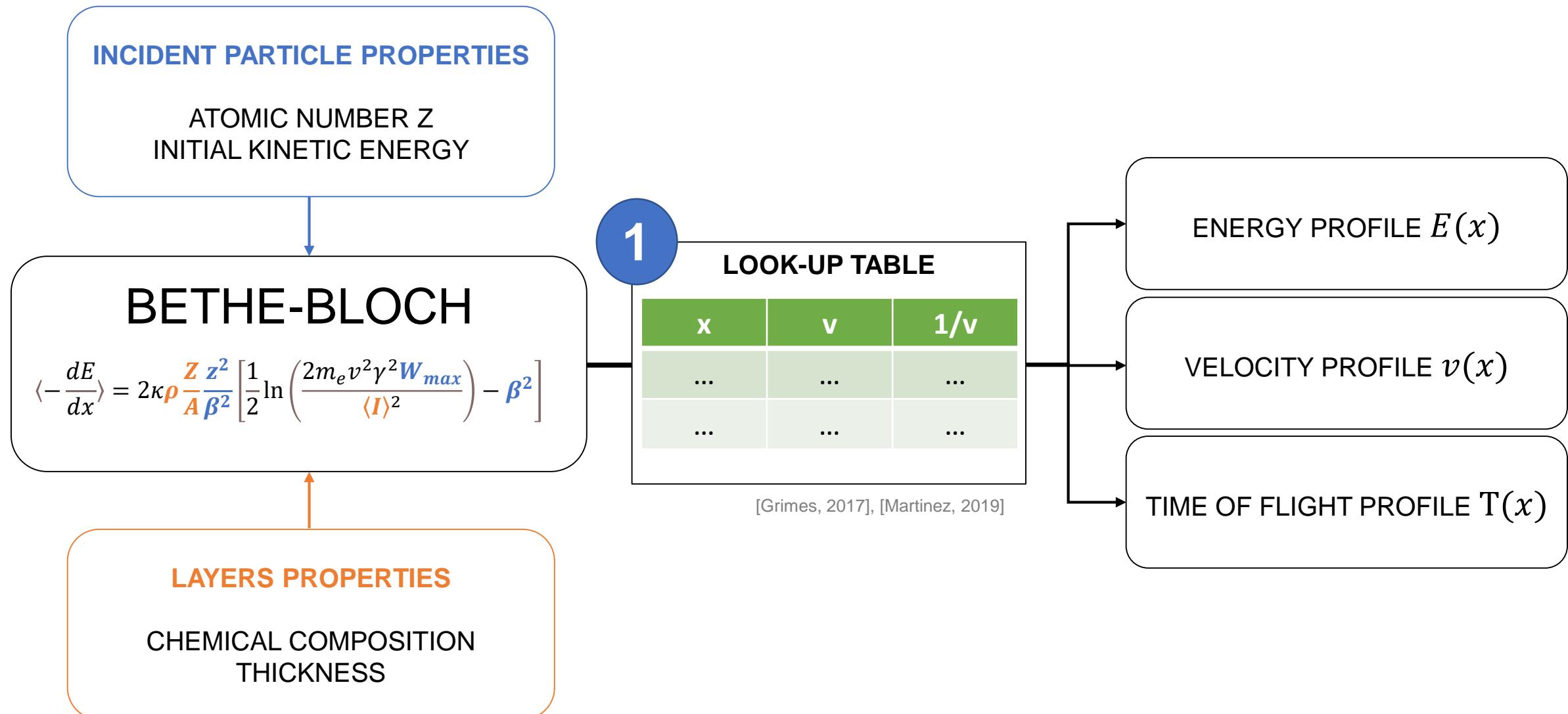


$$\left\langle -\frac{dE}{dx} \right\rangle = 2\kappa \rho \frac{Z z^2}{A \beta^2} \left[\frac{1}{2} \ln \left(\frac{2m_e v^2 \gamma^2 W_{max}}{\langle I \rangle^2} \right) - \beta^2 \right]$$

stopping power for **HEAVY CHARGED PARTICLES**

$0.1 \leq \beta\gamma \leq 1000$
traversing a prior-defined **MEDIUM**

The Bethe-Bloch algorithm



Bethe-Bloch approximate solution

1

$$\left\langle -\frac{dE}{dx} \right\rangle = 2\kappa z^2 \rho \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \left(\frac{2m_e c^2 \beta^2 \gamma}{\langle I \rangle} \right) - \beta^2 \right]$$

$$\begin{aligned} E &= (\gamma - 1)m_p c^2 \\ \frac{dE}{dx} &= \frac{dE}{dv} \frac{dv}{dx} = m_p v(x) \gamma^3 \frac{dv}{dx} \\ dx' &= \gamma^3 dx \\ \frac{dE}{dx'} &= m_p v(x') \frac{dv}{dx'} \\ A &= \frac{2}{m_p} \kappa z^2 \frac{Z}{A} \rho c^2 \quad [m^3/s^4] \end{aligned}$$

$$\begin{aligned} \ln \left(\frac{2m_e c^2 \beta^2 \gamma}{\langle I \rangle} \right) - \beta^2 &\approx \ln \left(\frac{2m_e c^2 \beta^2}{\langle I \rangle} \right) \\ B &= \frac{2m_e}{\langle I \rangle} \quad [s^2/m^2] \end{aligned}$$

$$-\nu \frac{dv}{dx'} = \frac{A}{v^2} \ln(Bv^2)$$

$$\begin{aligned} l_c &= v_0^4/A, \beta_0 = v_0/c, b = Bv_0^2 \\ \nu &= \frac{\nu}{v_0}, x = \frac{x}{l_c} \end{aligned}$$

$$\frac{dx}{d\nu} = -\frac{\nu^3}{(1 - (\beta_0 \nu)^2)^{\frac{3}{2}}} (ln(b\nu^2))^{-1} \equiv F(\nu)$$

$F(\nu)$ can be approximated by a Taylor series about $\beta_0 = 0$

Bethe-Bloch approximate solution

$$\frac{dx}{dv} = F(v) = -\frac{v^3}{\ln(bv^2)} - \beta_0^2 \frac{3v^5}{2\ln(bv^2)} - \beta_0^4 \frac{v^7}{\ln(bv^2)} \left(\frac{15}{8} - \frac{1}{2\ln(bv^2)} \right) - \beta_0^6 \frac{v^9}{\ln(bv^2)} \left(\frac{35}{16} - \frac{13}{12\ln(bv^2)} \right) + O(\beta_0^8)$$

Taylor series about $\beta_0 = 0$

... seeking a solution like: $x(v) = x_0(v) + \beta_0^2 x_1(v) + \beta_0^4 x_2(v) + \beta_0^6 x_3(v) + O(\beta_0^8)$

$$x - R = -v^4 \left(\frac{2 \ln^2 bv^2 + \ln bv^2 + 1}{8 \ln^3 bv^2} \right) - \beta_0^2 v^6 \left(\frac{9 \ln^2 bv^2 + 3 \ln bv^2 + 2}{36 \ln^3 bv^2} \right) + \\ - \beta_0^4 v^8 \left(\frac{120 \ln^2 bv^2 - 2 \ln bv^2 - 1}{512 \ln^3 bv^2} \right) + 31 \beta_0^6 v^{10} \left(\frac{25 \ln^2 bv^2 + 5 \ln bv^2 + 2}{2400 \ln^3 bv^2} \right) + O(\beta_0^8)$$

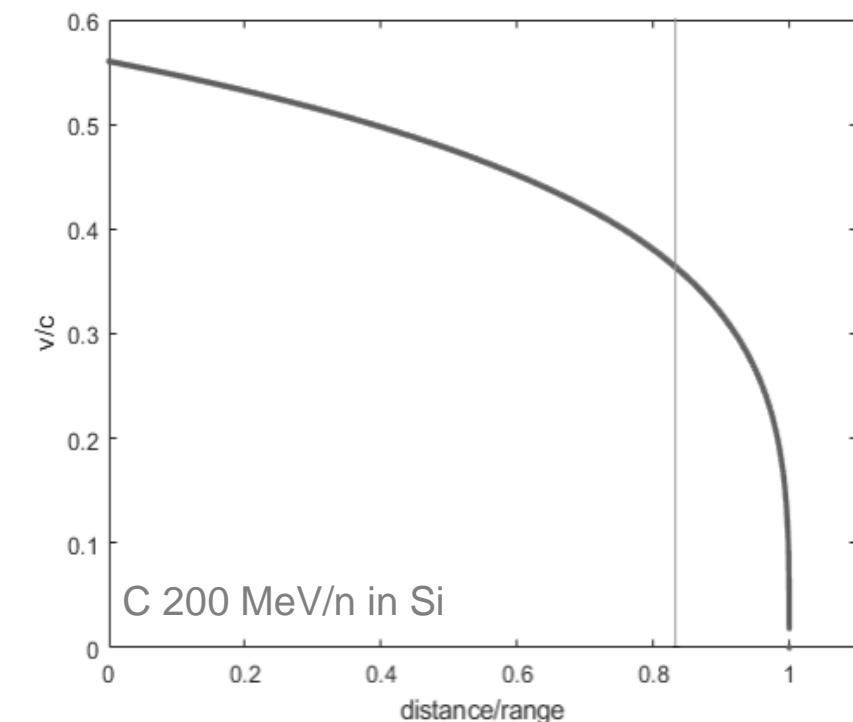
[Grimes, 2017], [Martinez, 2019]

BETHE-BLOCH

$$\langle -\frac{dE}{dx} \rangle = 2\kappa \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\frac{1}{2} \ln \left(\frac{2m_e v^2 \gamma^2 W_{max}}{\langle I \rangle^2} \right) - \beta^2 \right]$$

LOOK-UP TABLE

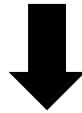
x	v	1/v
...
...



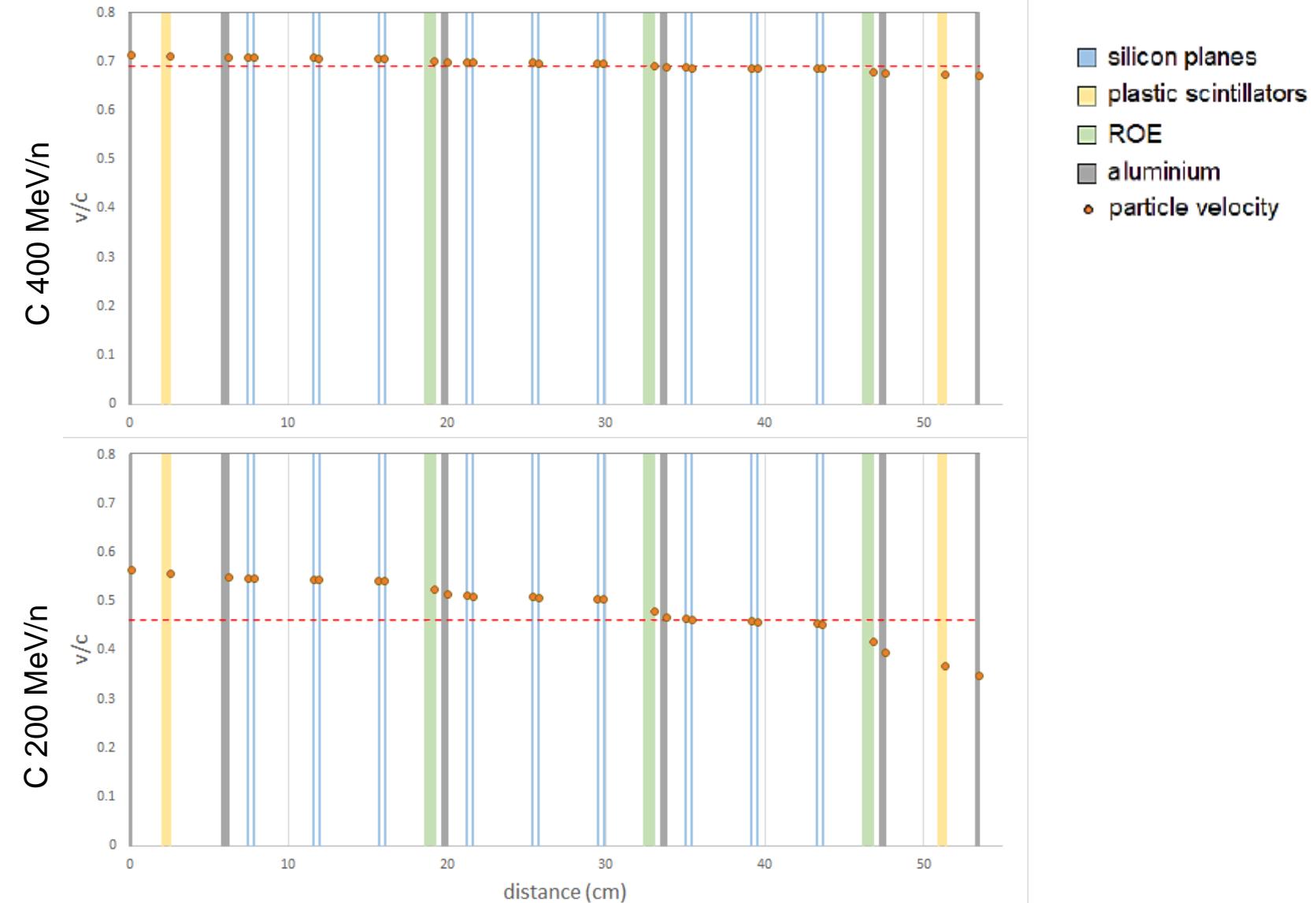
Velocity profile $v(x)$

mean velocity:

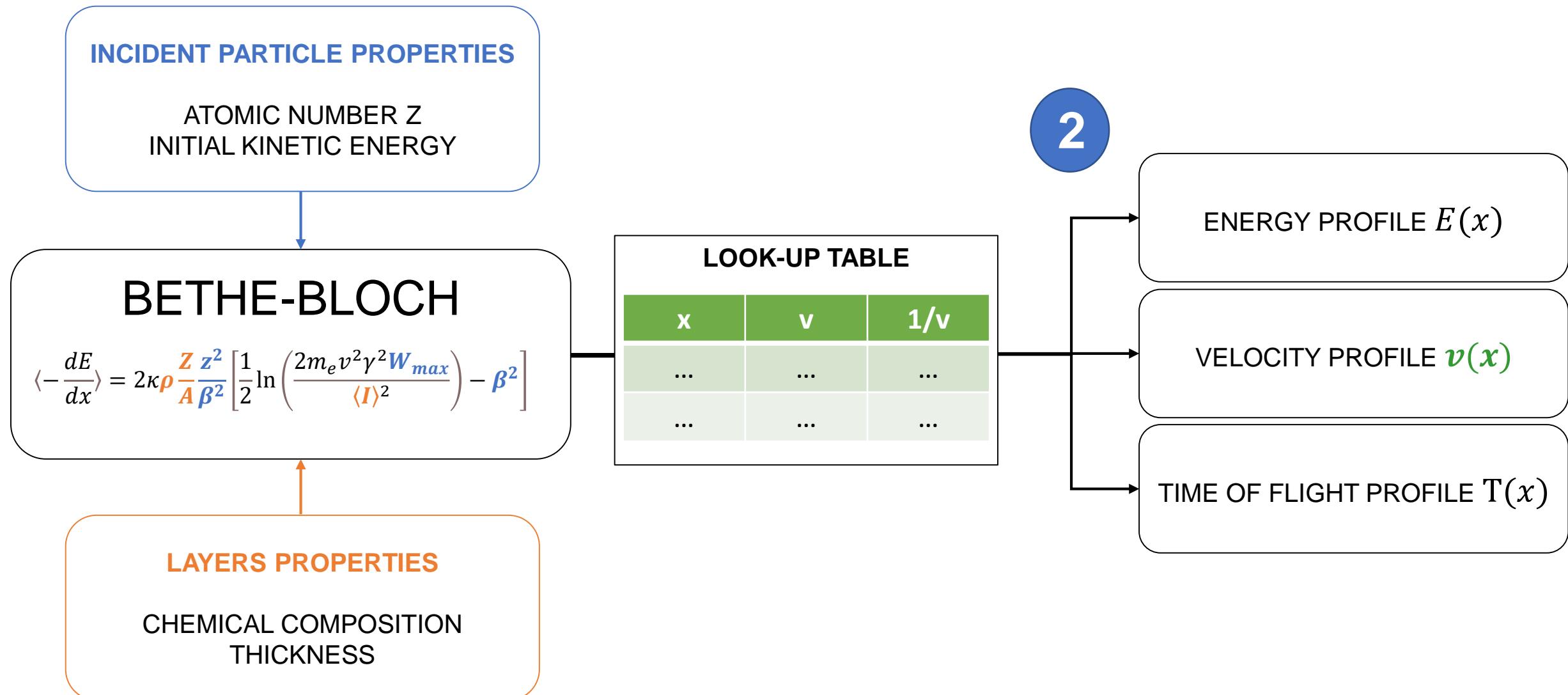
$$v_m = \frac{v_{in} + v_{fin}}{2} = \frac{\Delta s}{TOF}$$



VELOCITY PROFILE $v(x)$



The Bethe-Bloch algorithm

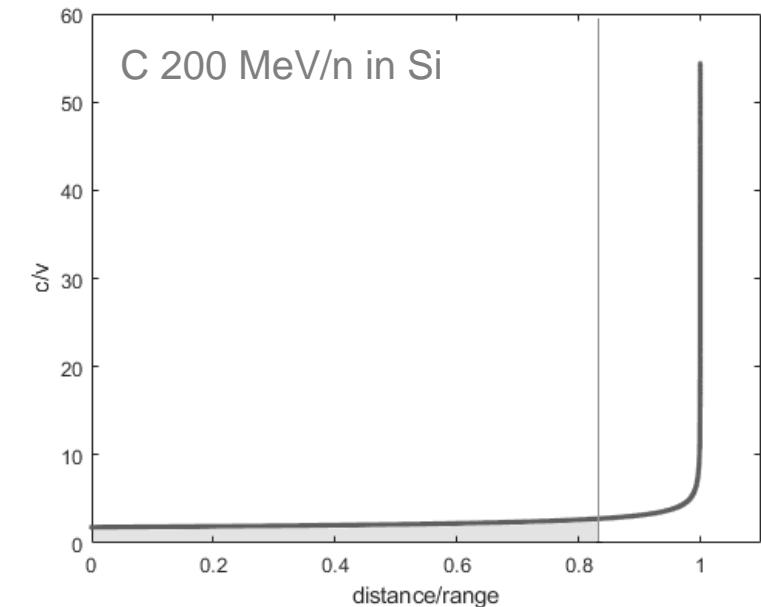
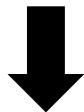


2

$$\Delta T = \int_0^{\Delta x} \frac{1}{v(x)} dx$$

$$E = \int_0^{\Delta x} \left\langle -\frac{dE}{dx} \right\rangle dx = \kappa z^2 \rho \frac{Z c^2}{A 2} \int_0^{\Delta x} \frac{f(v(x)/c)}{v(x)^2} dx$$

$$f(\beta) = \ln \left(\frac{2m_e c^2 \beta^2}{\langle I \rangle} \right)$$



$$TOF = \sum_{i=1}^m \Delta T_m = \sum_{i=1}^m \int_0^{\Delta x_m} \frac{1}{v_m(x)} dx = \sum_{i=1}^m \sum_{i=0}^{n_m-1} \frac{(x_{i+1} - x_i)}{2} \left(\frac{1}{v_m(x_{i+1})} + \frac{1}{v_m(x_i)} \right)$$

$$E = \sum_{i=1}^m \Delta E_m = \sum_{i=1}^m \kappa z^2 \rho \frac{Z c^2}{A 2} \int_0^{\Delta x_m} \frac{f(v(x)/c)}{v_m(x)^2} dx = \kappa z^2 \rho \frac{Z c^2}{A 2} \sum_{i=1}^m \sum_{i=0}^{n_m-1} \frac{(x_{i+1} - x_i)}{2} \left(\frac{f(v(x_{i+1})/c)}{v_m(x_{i+1})^2} + \frac{f(v(x_i)/c)}{v_m(x_i)^2} \right)$$

Straggling

STRAGGLING FUNCTION GIVEN BY LANDAU:

$$f(t, \Delta, \delta_2) = \frac{1}{\pi E_M} e^{k(1+\beta^2 \Gamma)} \int_0^\infty e^{kf_1(y) - \frac{k\delta_2 y^2}{2KE_M}} \cos(\lambda_1 y + kf_2(y)) dy$$

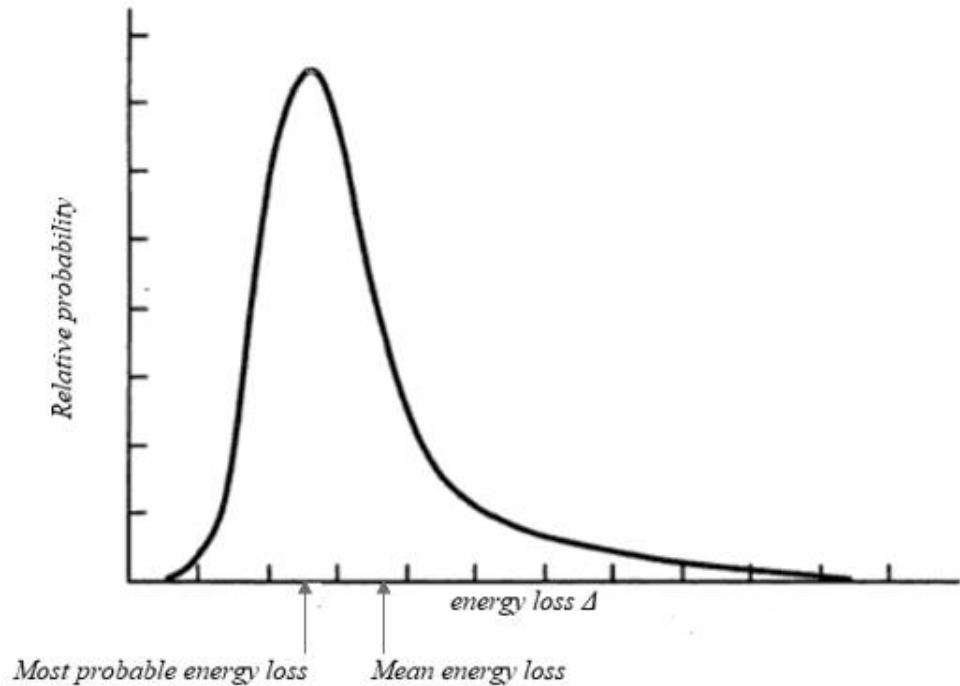
$k = \frac{\xi}{E_M}$

- for thin absorbers, $k < 10$
- for $k \geq 10$, the Landau distribution approaches the Gaussian limit

$$\xi = 2.55 \times 10^{-19} N_0 Z t \frac{Z^2}{\beta^2} \quad \text{Landau parameter [KeV]}$$

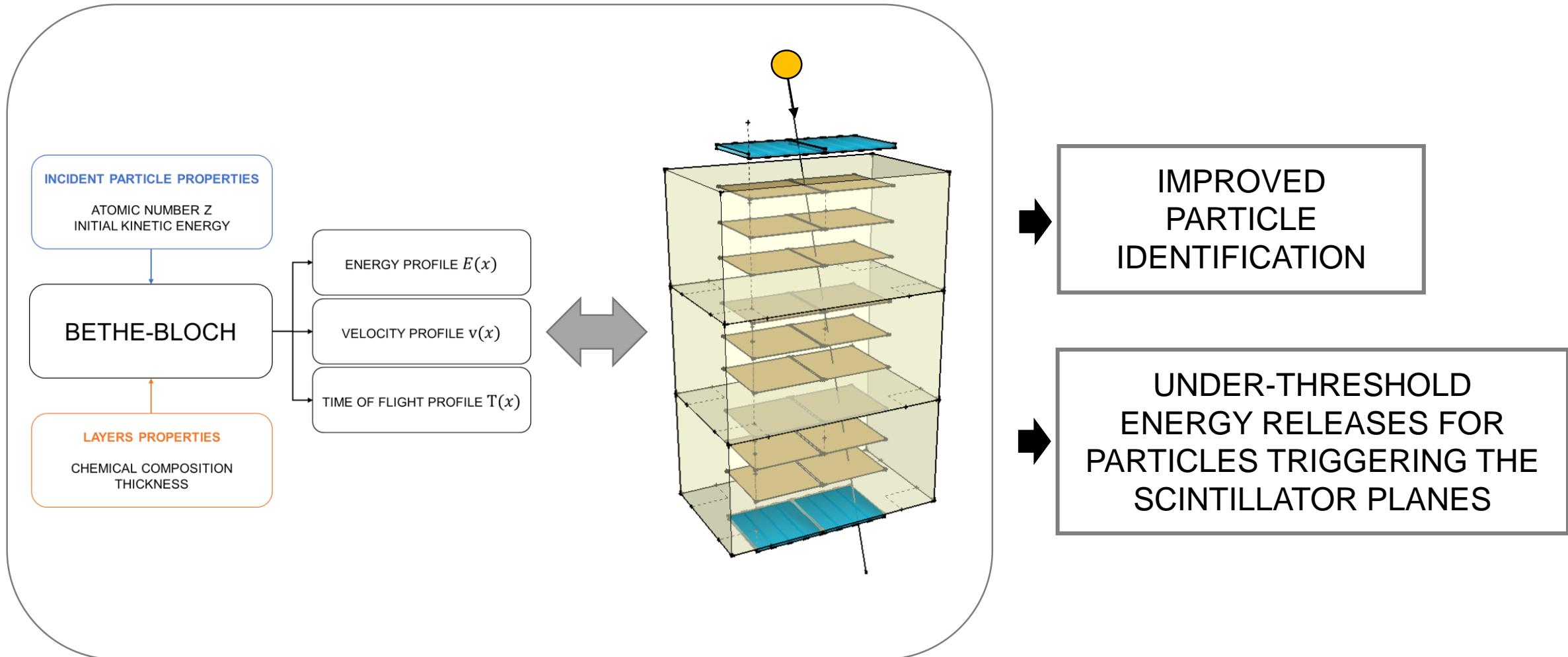
$$\langle \Delta \rangle = 2B\xi \quad \text{mean energy loss}$$

$\omega_L = 4.018 \xi \quad \text{FWHM for Landau function}$



[Bichsel, 1988]

Results from LIDAL

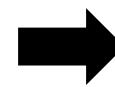
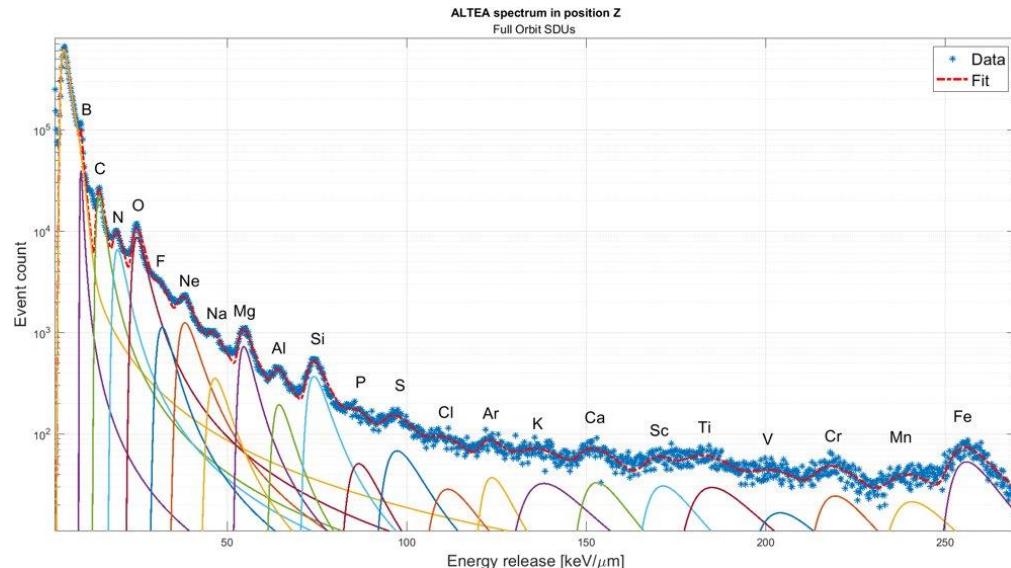


Particle identification

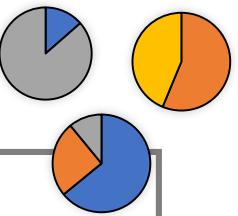


hybrid candidates
for detected particles

ALTEA SPECTRUM + FIT WITH LANDAU FUNCTIONS



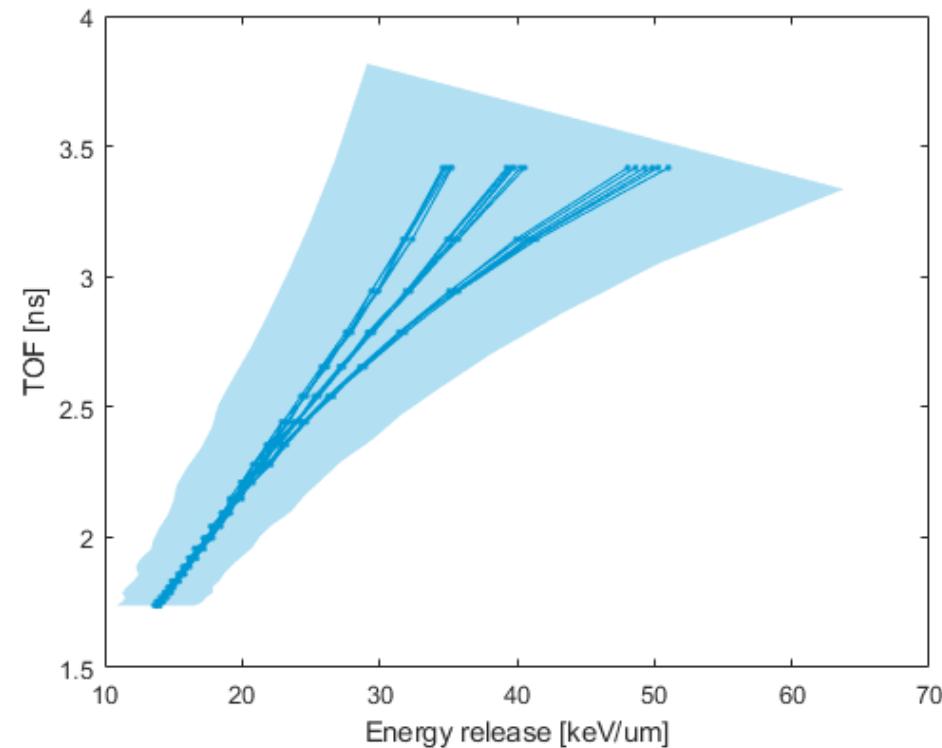
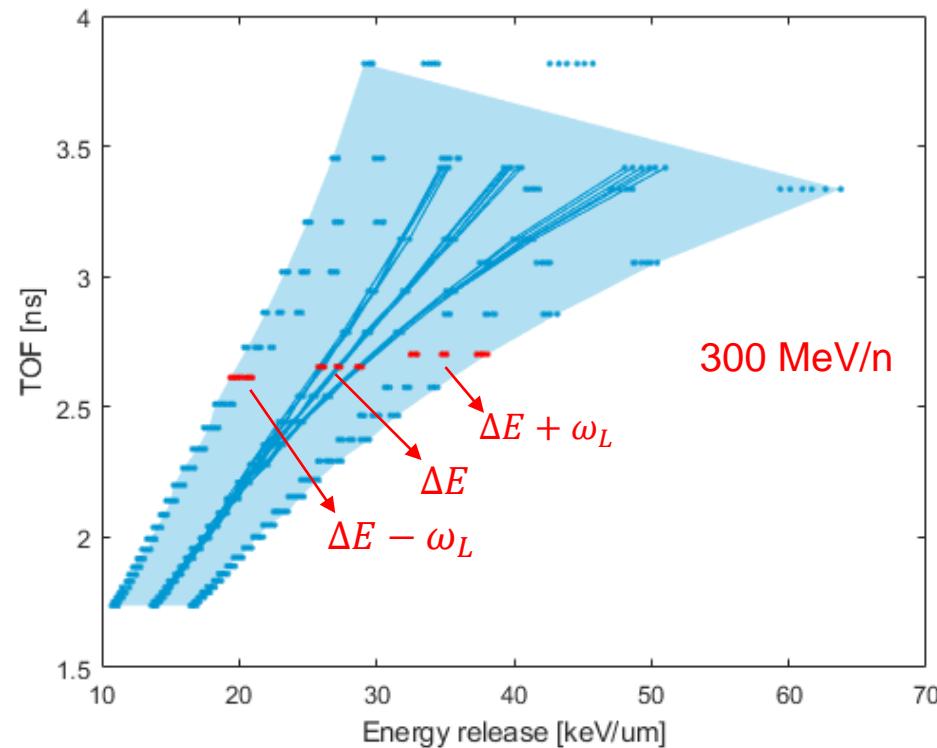
SINGLE
PARTICLE
IDENTIFICATION
FROM REFERENCE
SPECTRUM



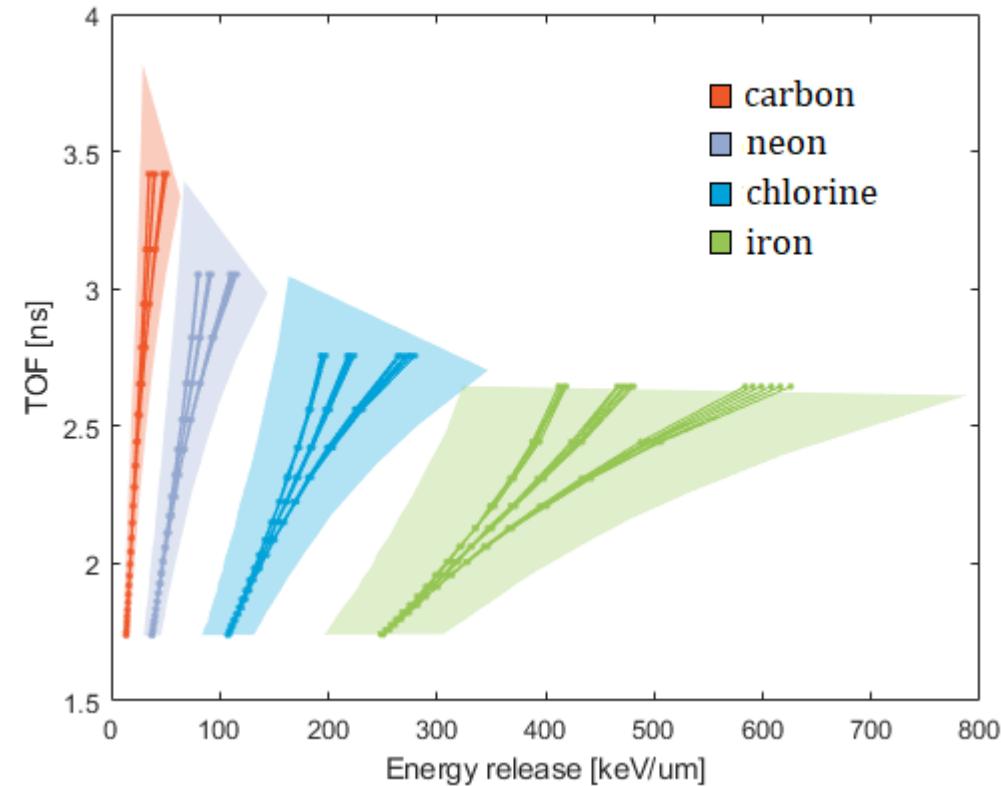
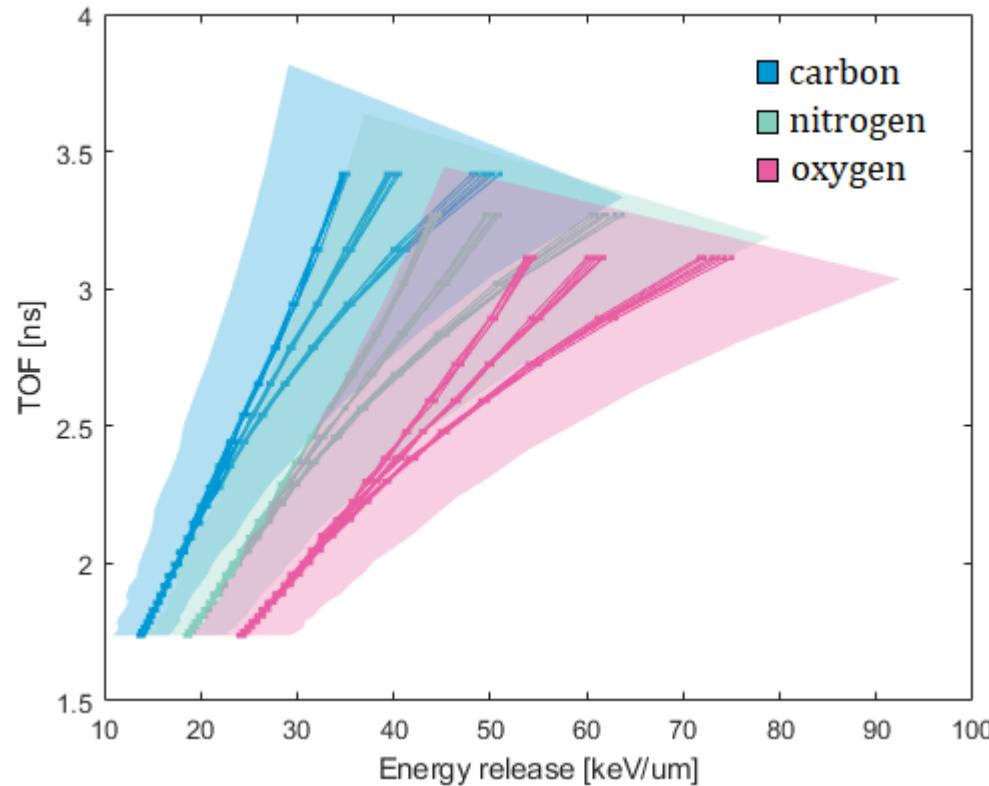
for more details: talk by G. Santi Amantini

Particle identification

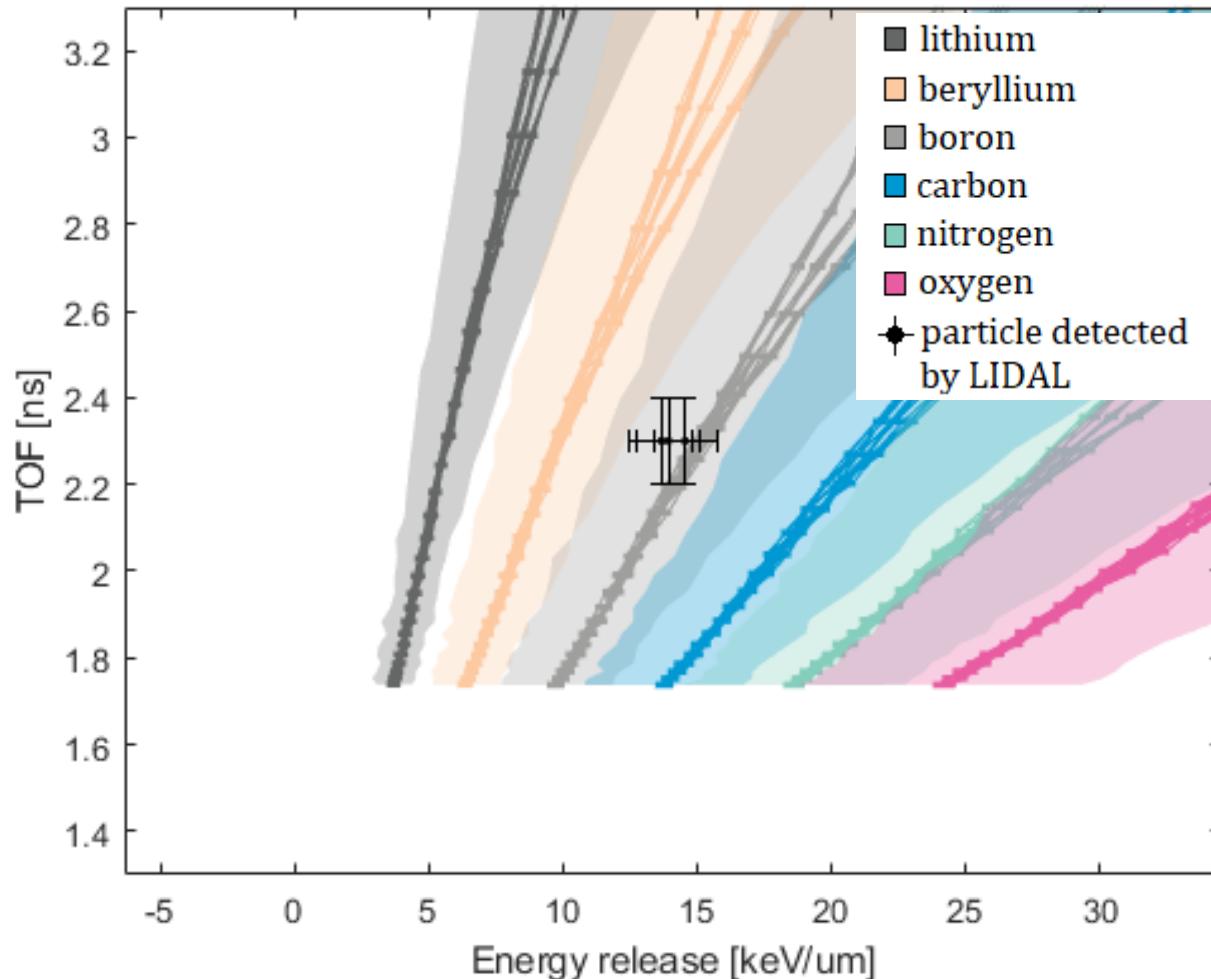
CARBON IONS



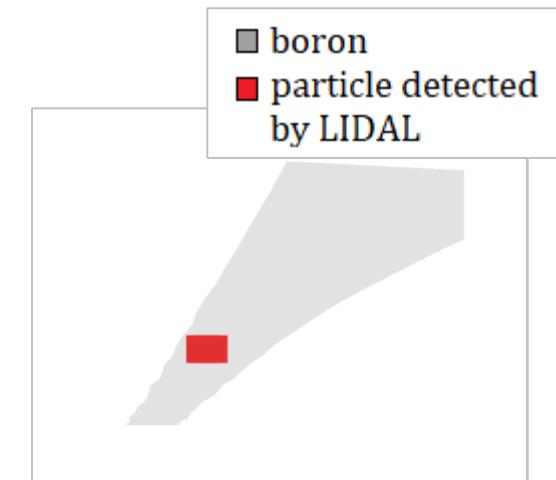
Particle identification



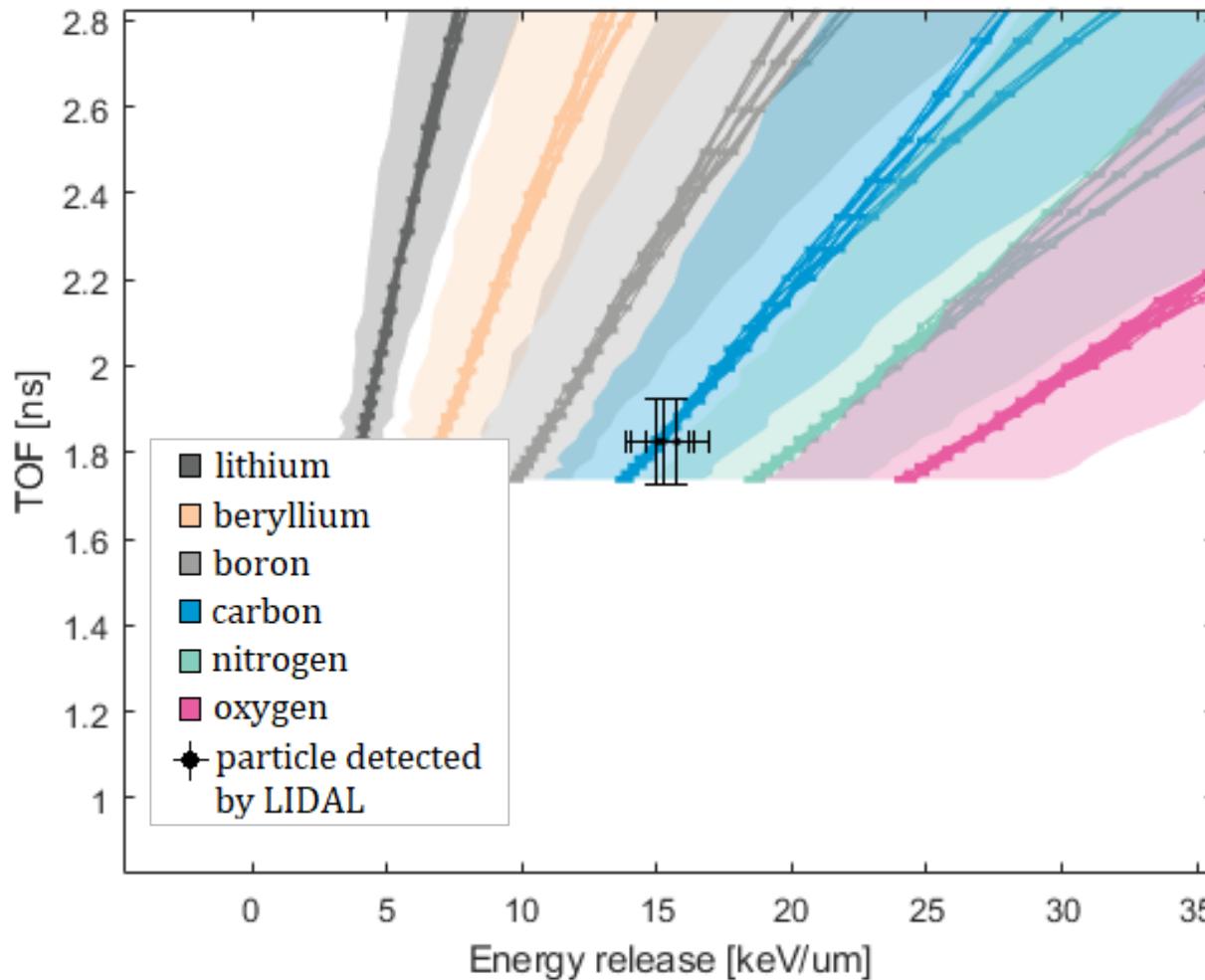
Particle identification



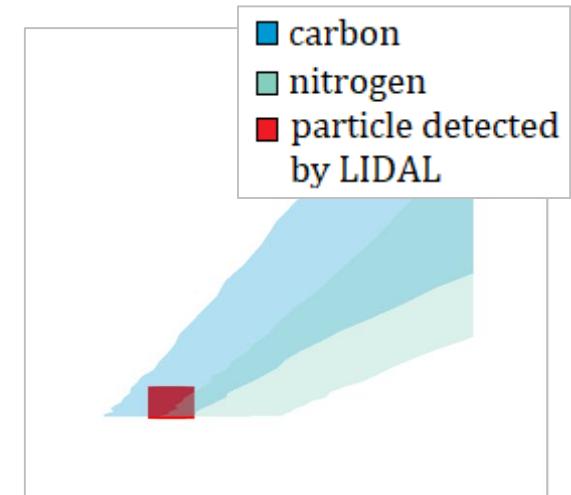
from Bethe-Bloch
algorithm:
**100% boron ion
at 500 MeV/n**



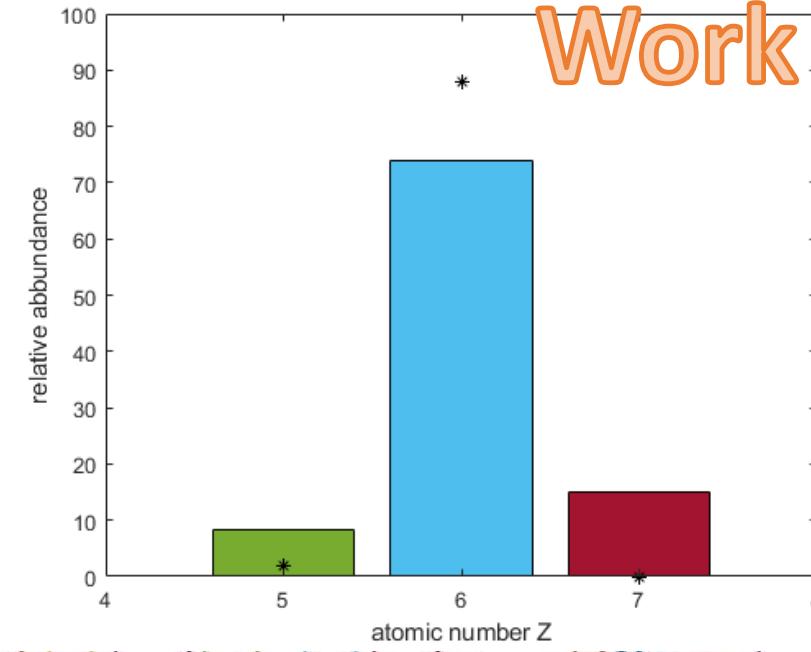
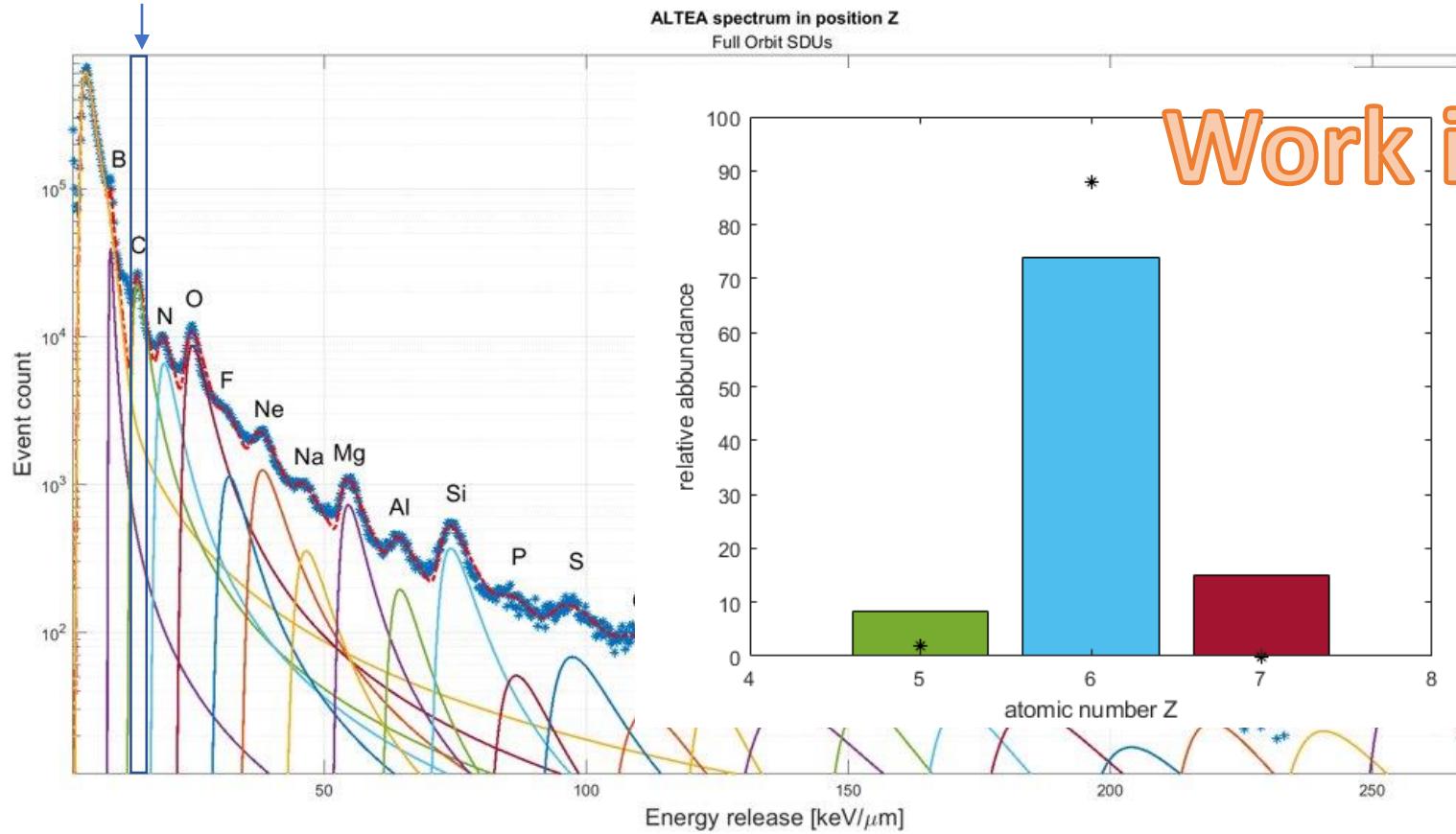
Particle identification



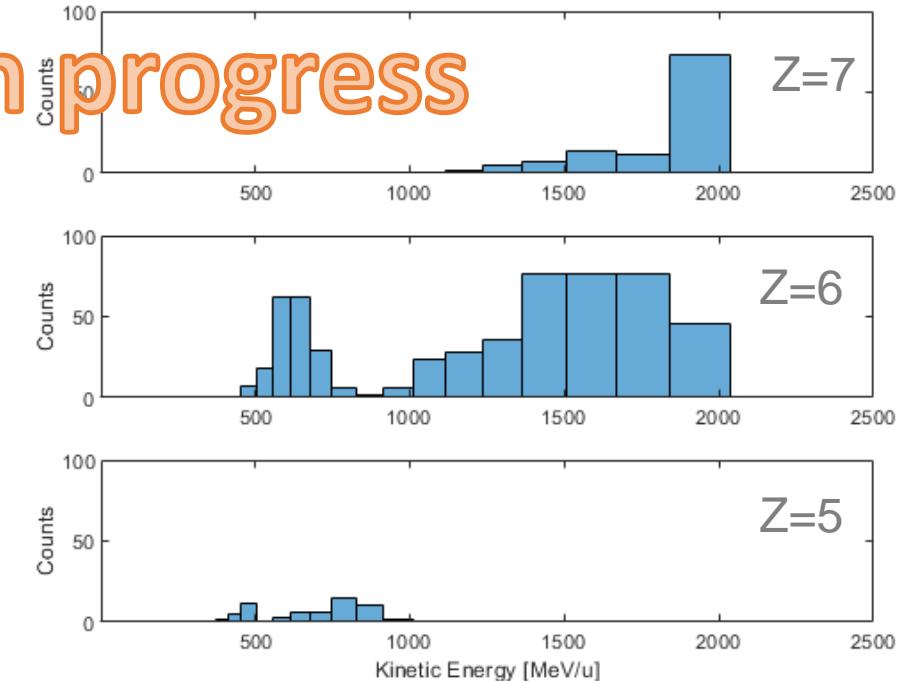
from Bethe-Bloch algorithm:
70% carbon ion at 1.2 GeV/n
30% nitrogen ion at 1.7 GeV/n



Particle identification



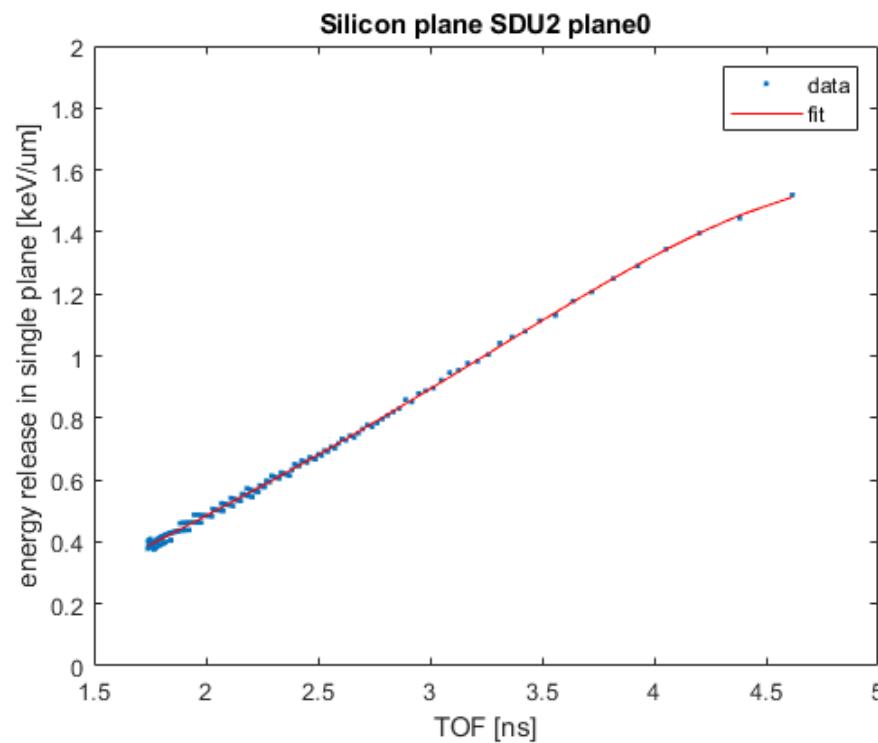
Work in progress



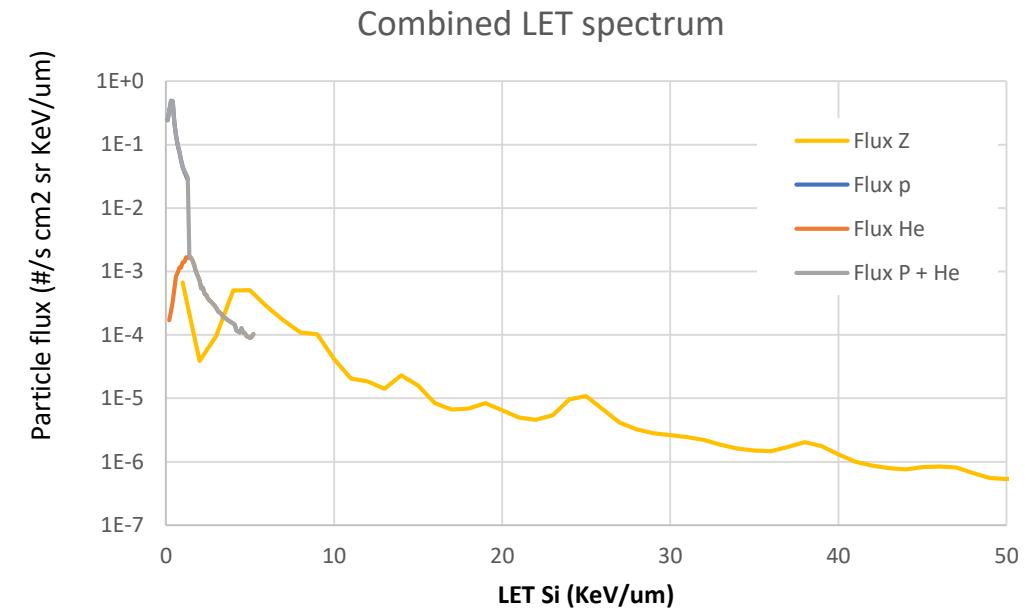
analysis from 750 particles detected by LIDAL

Under-threshold energy releases

$$E = p_1 \text{TOF}^4 + p_2 \text{TOF}^3 + p_3 \text{TOF}^2 + p_4 \text{TOF} + p_5$$



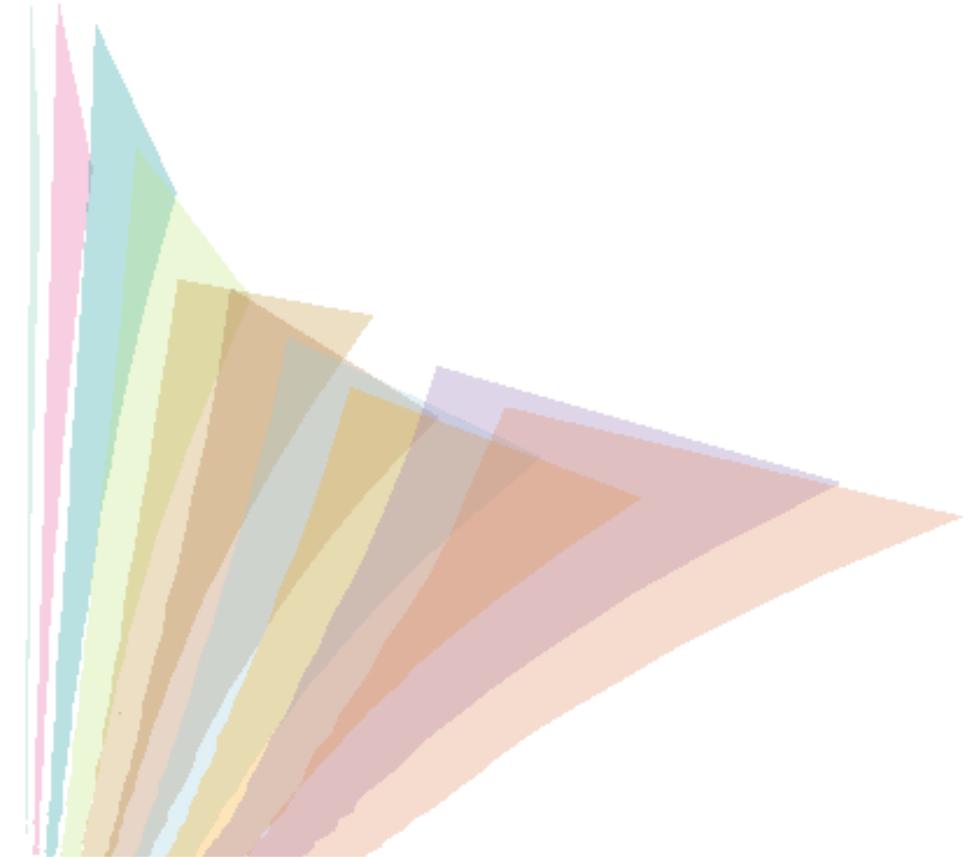
Work in progress



for more details: talk by L. Di Fino

Future perspectives

- improve the outlined method with MonteCarlo simulations
- extend the analysis to all particles detected by LIDAL so far



The LIDAL team



*L. Narici
L. Di Fino
G. Romoli
G. Santi Amantini
L. Lunati
V. Boretti*