



Workshop on Radiation Monitoring for the International Space Station (WRMISS)

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**DDFRG: DOUBLE DIFFERENTIAL FRAGMENTATION MODELS  
FOR LIGHT PARTICLE PRODUCTION IN NUCLEAR REACTIONS  
FOR SPACE RADIATION PROTECTION**

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# OUTLINE

- 1 INTRODUCTION
- 2 PROTON MODEL
- 3 COALESCENCE SCALING
- 4 LIGHT ION MODEL
- 5 PION MODEL
- 6 NEUTRON MODEL
- 7 SUMMARY, CONCLUSIONS AND FUTURE WORK

# 1. INTRODUCTION

- **HZETRN** (High charge (Z) and Energy TRaNsport) is NASA's primary space radiation transport code
- Primary engine behind **OLTARIS** (On-Line Tool for the Assessment of Radiation In Space) and **NAIRAS** (Nowcast of Aerospace Ionizing Radiation System). Pre-computed results also used in **ARRT** (Acute Radiation Risks Tool).
- HZETRN is very fast, deterministic code capable of efficient analysis of a wide variety of human space mission scenarios
- Extensively benchmarked against the majority of the world's other radiation transport codes - agrees with world codes as well as they agree with each other
- Many nuclear physics codes (**DDFRG**, **NUCFRG**, **EMDFRG**, **RAADFRG**, **NUCDAT**) have been developed "in-house" in order to enable fast and efficient calculations with HZETRN
- 3DHZETRN is the most recent development, which is fully 3-dimensional (3D)
- Neutrons, protons, light ions ( $^2,3\text{H}$ ,  $^3,4\text{He}$ ),  $\pi$  account for large fraction of radiation dose - Because they are light, they scatter at large angles and require a 3D treatment
- **DDFRG** (Double-Differential (DD) FRaGmentation) - new nuclear physics code which calculates double-differential cross sections for neutron, proton, light ion,  $\pi$  production - Provides closed-form, analytic formulas - highly efficient

# INTRODUCTION - PUBLICATIONS

## Journal Publications:

- 1 *DDFRG2: Double-Differential FRaGmentation model for pion production in high energy nuclear collisions*, J. Norbury  
Nuclear Instruments and Methods in Physics Research A, vol. 1053, p. 168336, 2023
- 2 *Double-Differential FRaGmentation (DDFRG) models for proton and light ion production in high energy nuclear collisions*, J. Norbury  
Nuclear Instruments and Methods in Physics Research A, vol. 986, p. 164681, 2021
- 3 *Light ion double-differential cross section parameterizations and results from the SHIELD transport code*, J. Norbury, L. Latysheva, N. Sobolevsky  
Nuclear Instruments and Methods in Physics Research A, vol. 947, p. 162576, 2019

## NASA Technical Publications: can be found at <http://ntrs.nasa.gov/>

- 4 *DDFRG2: Double-Differential FRaGmentation model with pion production*, J. Norbury  
NASA Technical Publication 2022001664, 2023
- 5 *Double-Differential FRaGmentation (DDFRG) models for proton and light ion production in high energy nuclear collisions valid for both small and large angles*, J. Norbury  
NASA Technical Publication 2020-5001740, 2020
- 6 *Light ion double-differential cross sections for space radiation*, J. Norbury  
NASA Technical Publication 2018-220077, 2018



## 2. PROTON MODEL (DDFRG1)

- Thermal proton production model
  - Protons being produced from four separate sources:
  - Projectile, Target, Central fireball, Direct projectile knockout
  - *Source models done before but parameters fitted to each individual experiment*
  - *New model global fit to all data with single set of parameters*

### PROTON THERMAL MODEL FOR *each* SOURCE

$$E \frac{d^3\sigma}{dp^3} = N e^{-T/\Theta}$$

- $N$  determined from requirement that integral of double differential cross section gives total cross section, and  $\sigma$

# PROTON MODEL

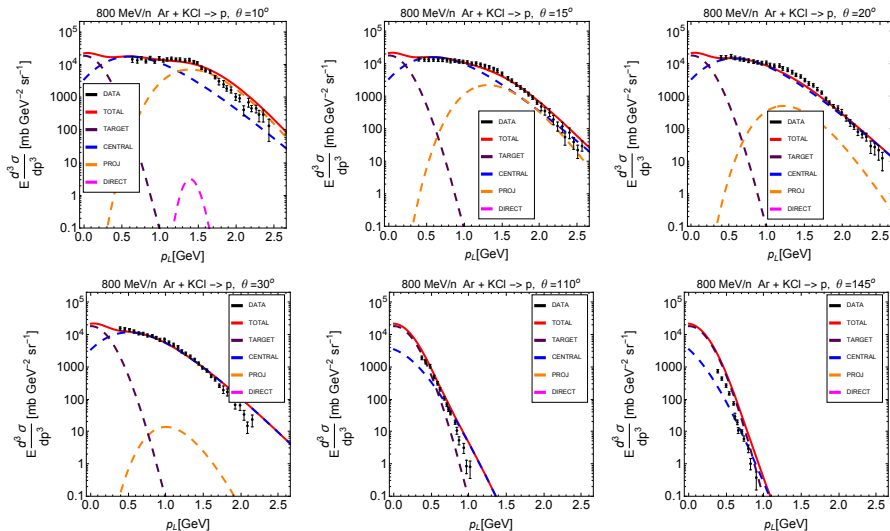
- In terms of total 3-momentum and angle variables ( $p_{jL}, \theta_{jL}$ ) the final thermal plus direct knockout model is

## DDFRG PROTON THERMAL MODEL

$$\begin{aligned} E \frac{d^3\sigma}{dp^3}(p_{jL}, \theta_{jL}) = & \frac{\sigma}{4\pi m} \{ \exp[-(\gamma_{\mathcal{P}L} \sqrt{p_{jL}^2 + m^2} - \gamma_{\mathcal{P}L} \beta_{\mathcal{P}L} p_{jL} \cos \theta_{jL} - m)/\Theta_{\mathcal{P}}] \\ & + \exp[-(\gamma_{\mathcal{C}L} \sqrt{p_{jL}^2 + m^2} - \gamma_{\mathcal{C}L} \beta_{\mathcal{C}L} p_{jL} \cos \theta_{jL} - m)/\Theta_{\mathcal{C}}] \\ & + \exp[-(\gamma_{\mathcal{T}L} \sqrt{p_{jL}^2 + m^2} - \gamma_{\mathcal{T}L} \beta_{\mathcal{T}L} p_{jL} \cos \theta_{jL} - m)/\Theta_{\mathcal{T}}] \\ & + w_{\mathcal{D}}^{(p)} \exp[-(\gamma_{\mathcal{P}L} \sqrt{p_{jL}^2 + m^2} - \gamma_{\mathcal{P}L} \beta_{\mathcal{P}L} p_{jL} \cos \theta_{jL} - m)/\Theta_{\mathcal{D}}] \} \\ \times & \left\{ \Theta_{\mathcal{P}} e^{m/\Theta_{\mathcal{P}}} K_1(m/\Theta_{\mathcal{P}}) + \Theta_{\mathcal{C}} e^{m/\Theta_{\mathcal{C}}} K_1(m/\Theta_{\mathcal{C}}) + \Theta_{\mathcal{T}} e^{m/\Theta_{\mathcal{T}}} K_1(m/\Theta_{\mathcal{T}}) + w_{\mathcal{D}}^{(p)} \Theta_{\mathcal{D}} e^{m/\Theta_{\mathcal{D}}} K_1(m/\Theta_{\mathcal{D}}) \right\}^{-1} \end{aligned}$$

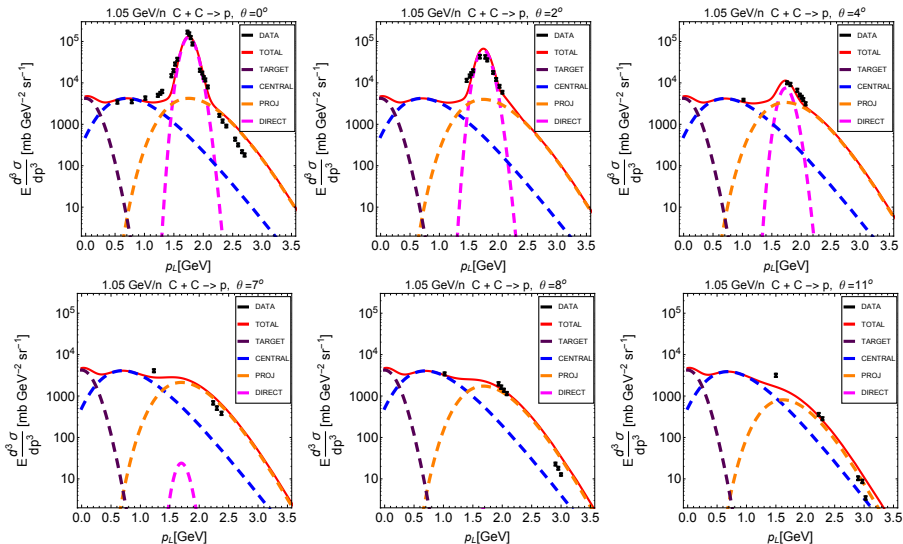
- Can be analytically integrated to give closed form analytic formula for  $\frac{d\sigma}{dE}$  (see Publications)

# LARGE ANGLE DDFRG 800 MeV/n Ar + KCl $\rightarrow$ p



**DDFRG model** agrees well with data, but some differences

# SMALL ANGLE DDFRG 1.05 GeV/n C + C → p



**DDFRG model** agrees well with data, but some differences

### 3. COALESCENCE SCALING

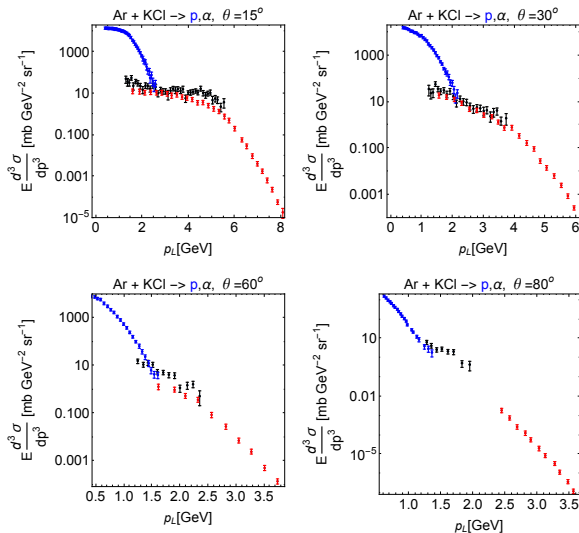
- Comparisons of double-differential cross section data between proton production and light ion production show that light ion data is very well represented by scaling proton data, assuming that light ions are produced via coalescence
- Comparisons done only using experimental proton data versus light ion data - no theoretical model used in comparing proton data to light ion data, except for simple scaling of proton data as

#### COALESCENCE MODEL

$$E_A \frac{d^3\sigma_A}{dp_A^3} = C_A \left( E_p \frac{d^3\sigma_p}{dp_p^3} \right)^A, \quad p_A \equiv Ap_p \text{ and } E_A \equiv AE_p$$

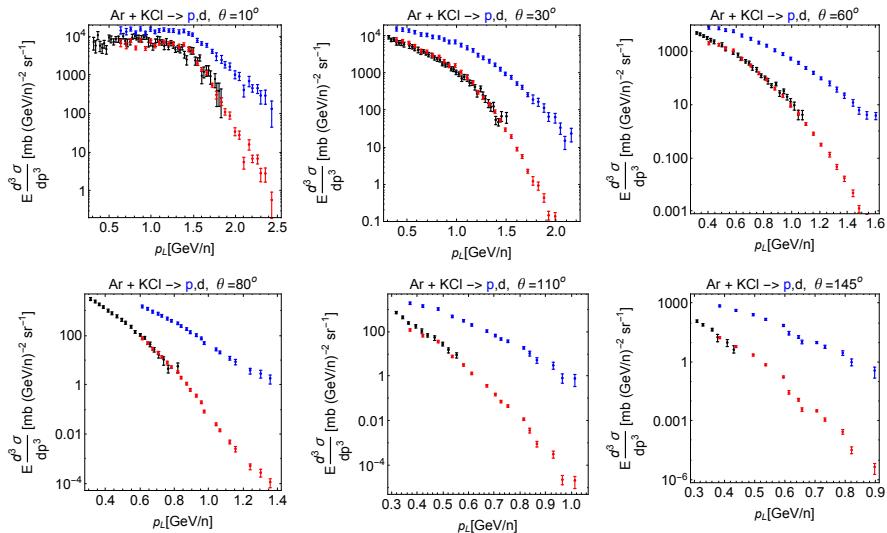
$C_A \equiv$  coalescence coefficient

- Coalescence  $\equiv$  Fusion (cf. Big Bang)



Proton data is **scaled** and then agrees very well with light ion data

# LARGE ANGLE SCALING - GEV/N UNITS: 800 MeV/n Ar + KCl $\rightarrow$ p, d

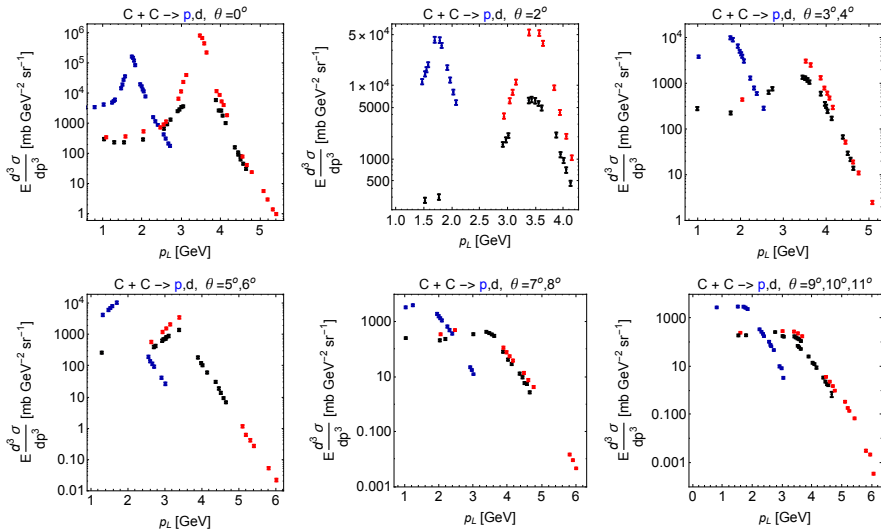


Proton data is **scaled** and then agrees very well with light ion data

- Data looks nothing like previous large angle data of Nagamiya  
- now a giant peak!
  
- Coalescence scaling fails!  
- at the smallest angles, but OK when angles increase



# SMALL ANGLE SCALING 1.05 GeV/n C + C → d



Proton data is scaled and then agrees with large angle light ion data, but not small angle

## 4. LIGHT ION MODEL (DDFRG1)

- Empirical relation between proton and light ion data implies not necessary for separate theoretical model for light ion production
- One only requires a model for proton production
- If proton model compares well to data, then scaled proton model will automatically compare well to light ion data
- Light ion model is obtained simply by scaling the proton model
- And this works for *all* composite light ions
- Separate light ion model not required
- However, the light ion model is not obtained without some effort
  - To demonstrate scaling of the experimental data, one typically uses a fitted coalescence coefficient,  $C_A$ , for each light ion
  - To develop a fully predictive model, this coefficient must be calculated with theoretical model developed separately

# LIGHT ION MODEL

$$E_A \frac{d^3 \sigma_A}{dp_A^3} = C_A \left\{ w_{\mathcal{P}} \left[ E \frac{d^3 \sigma}{dp^3}(\rho_{jL}, \theta_{jL}) \right]_{\mathcal{P}} + w_C \left[ E \frac{d^3 \sigma}{dp^3}(\rho_{jL}, \theta_{jL}) \right]_C + w_{\mathcal{T}} \left[ E \frac{d^3 \sigma}{dp^3}(\rho_{jL}, \theta_{jL}) \right]_{\mathcal{T}} + w_{\mathcal{D}} \left[ E \frac{d^3 \sigma}{dp^3}(\rho_{jL}, \theta_{jL}) \right]_{\mathcal{D}} \right\}^A$$

- Complicated *algebraic* expression when expanded out and written in terms of  $\sigma$  and Lorentz transformed to lab frame variables
- Left hand side is DD cross section for light ion production
- Right hand side contains DD cross sections for proton production
- Modified “coalescence” model

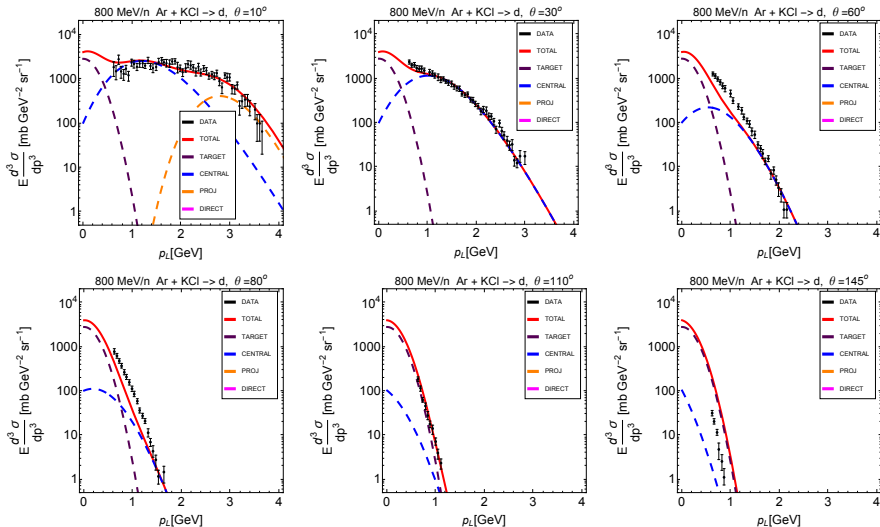
## THERMAL / COALESCENCE MODEL FOR LIGHT ION PRODUCTION

$$E_A \frac{d^3 \sigma_A}{dp^3_A} = C_A N_4^A \left\{ w_P \exp[(m_p - \gamma_{PL} \sqrt{p_{pL}^2 + m_p^2} + \gamma_{PL} \beta_{PL} p_{pL} \cos \theta_{pL}) / \Theta_P] \right. \\
+ w_C \exp[(m_p - \gamma_{CL} \sqrt{p_{pL}^2 + m_p^2} + \gamma_{CL} \beta_{CL} p_{pL} \cos \theta_{pL}) / \Theta_C] \\
+ w_T \exp[(m_p - \gamma_{TL} \sqrt{p_{pL}^2 + m_p^2} + \gamma_{TL} \beta_{TL} p_{pL} \cos \theta_{pL}) / \Theta_T] \\
\left. + w_D w_D^{(p)} \exp[(m_p - \gamma_{PL} \sqrt{p_{pL}^2 + m_p^2} + \gamma_{PL} \beta_{PL} p_{pL} \cos \theta_{pL}) / \Theta_D] \right\}^A$$

$$N_4 = \frac{\sigma_p}{4\pi m_p} \left[ \Theta_P e^{\frac{m_p}{\Theta_P}} K_1 \left( \frac{m_p}{\Theta_P} \right) + \Theta_C e^{\frac{m_p}{\Theta_C}} K_1 \left( \frac{m_p}{\Theta_C} \right) \right. \\
\left. + \Theta_T e^{\frac{m_p}{\Theta_T}} K_1 \left( \frac{m_p}{\Theta_T} \right) + w_D^{(p)} \Theta_D e^{\frac{m_p}{\Theta_D}} K_1 \left( \frac{m_p}{\Theta_D} \right) \right]^{-1}$$

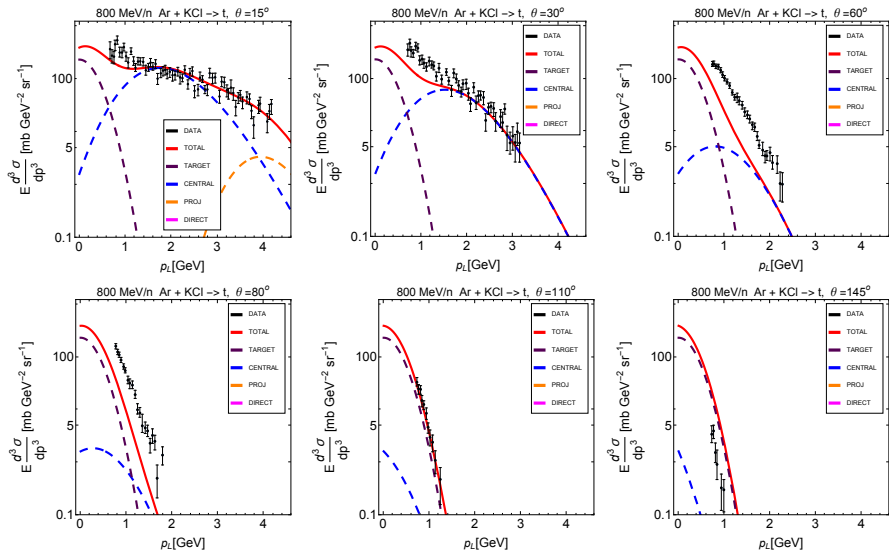
- Can be analytically integrated (for A = 2, 3, 4) to give closed form analytic formula for  $\frac{d\sigma}{dE}$  (see Publications)

# LARGE ANGLE DDFRG 800 MeV/n Ar + KCl $\rightarrow$ d



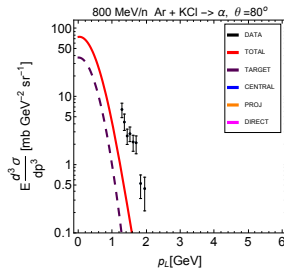
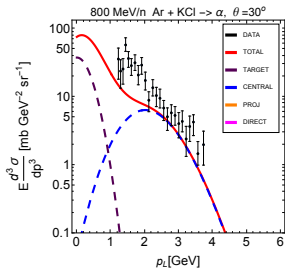
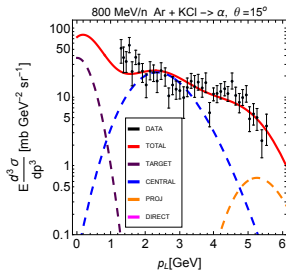
DDFRG model agrees well with data, but some differences

# LARGE ANGLE DDFRG 800 MeV/n Ar + KCl $\rightarrow$ t



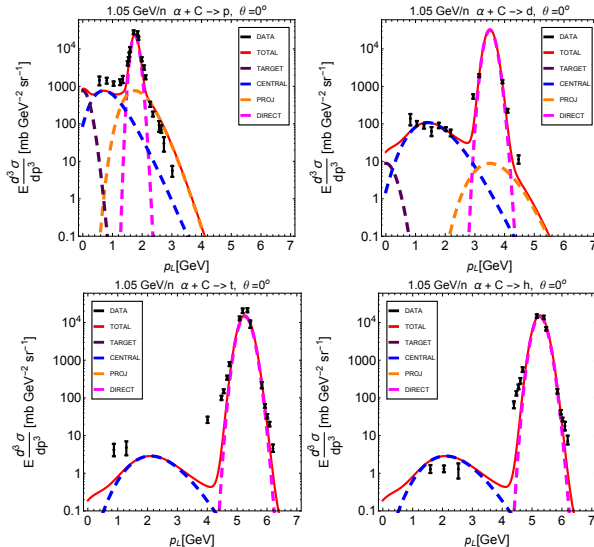
DDFRG model agrees well with data, but some differences

# LARGE ANGLE DDFRG 800 MeV/n Ar + KCl $\rightarrow \alpha$



DDFRG model agrees with  $15^\circ$  data, but differences at larger angles

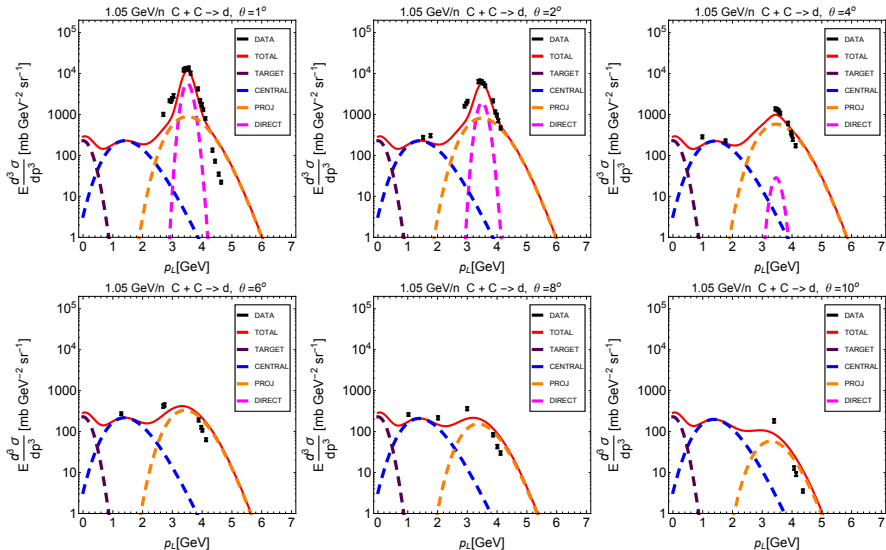
# SMALL ANGLE DDFRG 1.05 GeV/n $\alpha + C \rightarrow p, d, t, h$



DDFRG model agrees well with data, but some differences

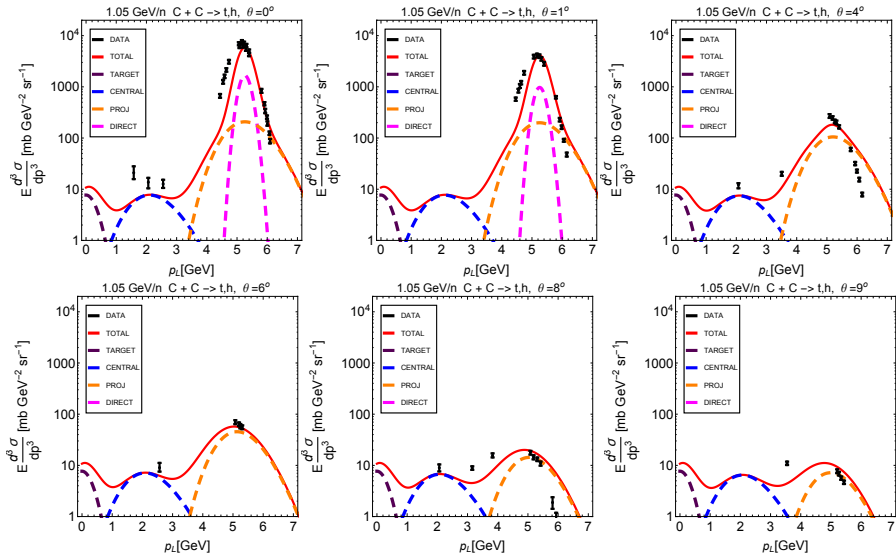


# SMALL ANGLE DDFRG 1.05 GeV/n C + C → d



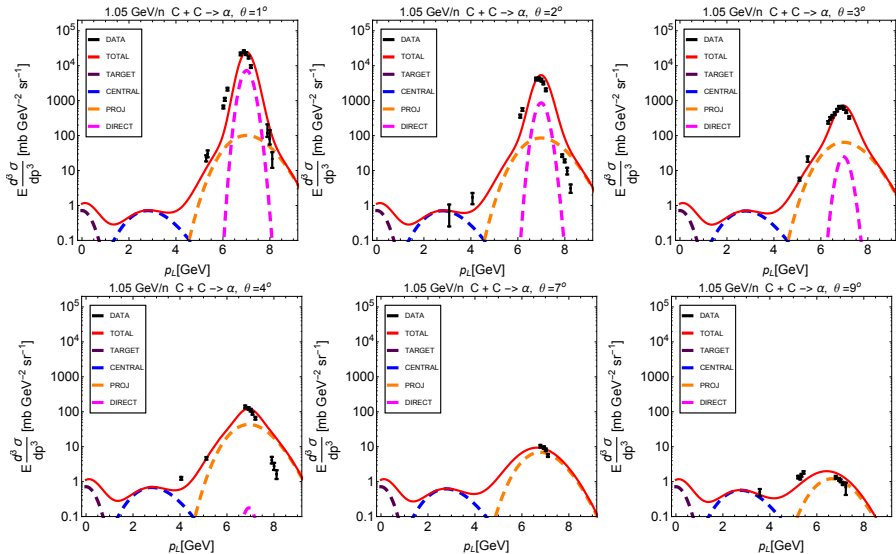
**DDFRG model** agrees well with data, but some differences

# SMALL ANGLE DDFRG 1.05 GeV/n C + C → t, h



**DDFRG model** agrees well with data, but some differences

# SMALL ANGLE DDFRG 1.05 GeV/n C + C → α



**DDFRG model** agrees well with data, but some differences

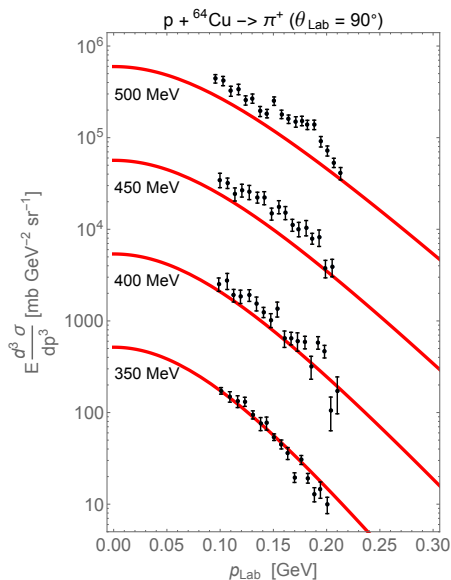
## 5. PION MODEL (DDFRG2)

- Just adapt Proton model?
  - Pions mainly produced from central hot fireball region
  - Simpler than previous models, because only one source
  - Just set all sources to zero, except central fireball?
- Significant complications arise because pion is so light  
Pion (140 MeV), Proton (938 MeV)
- **Complication # 1: Maximum pion energy and angle needs careful evaluation from relativistic kinematics**
  - Same true for nucleons and light ions, but no observable effects
  - Energy integral (0 to  $\infty$ ) modified
    - spectral distribution can no longer be obtained analytically
- **Complication # 2: Maximum pion energy cut-off**
  - This effect only showed up for one out of many reactions
  - Significant puzzle: Why only one reaction?

# PION MODEL (DDFRG2): FINAL MODEL DESCRIPTION

- Modify proton model
  - Change proton mass to pion mass
  - Change temperature parameters
- Total cross section normalization calculated from total pion cross section model [Norbury and Townsend, NIMB vol. 254, p.187 (2007)]
- Only one source  $\Rightarrow$  central fireball
- Maximum pion energy and angle incorporated = integration limits of total cross section normalization equations
- Maximum pion energy  $\Rightarrow$  cut-off incorporated into unique low energy reactions:  $p+A \rightarrow \pi^-$

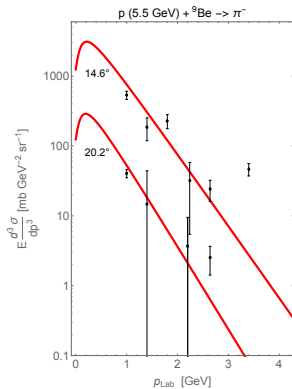
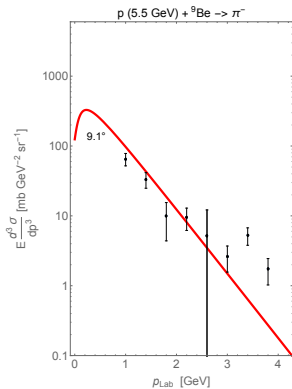
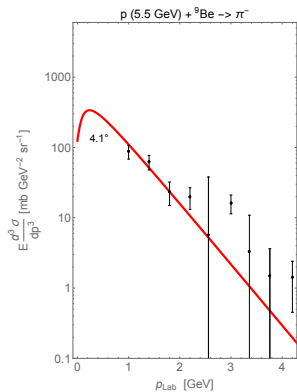
# PION MODEL RESULTS - LOW ENERGY PROTON PROJECTILES



Cross sections multiplied by increasing factors of 10

- DDFRG model agrees well with data, but some differences above 400 MeV

# PION MODEL RESULTS - HIGH ENERGY PROTON PROJECTILES

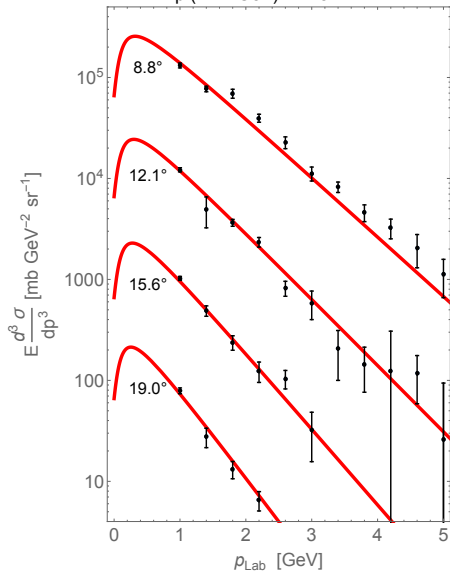


Cross section for 14.6° multiplied by 10

- DDFRG model agrees well with data

# PION MODEL RESULTS - HIGH ENERGY PROTON PROJECTILES

$p(11.4 \text{ GeV}) + {}^9\text{Be} \rightarrow \pi^-$

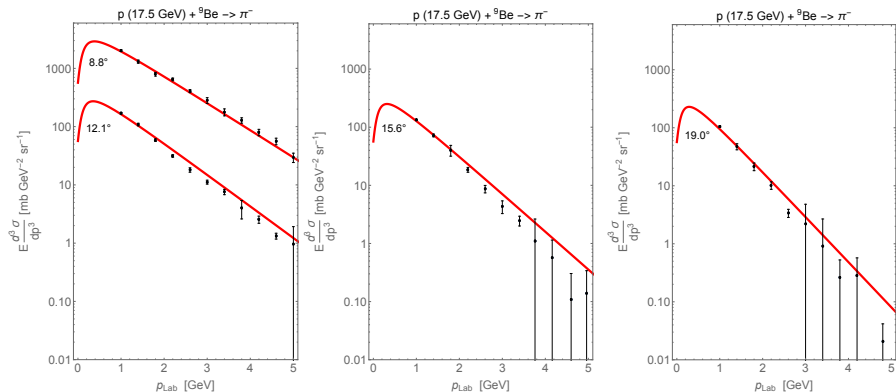


Cross sections multiplied by increasing factors of 10

- DDFRG model agrees well with data



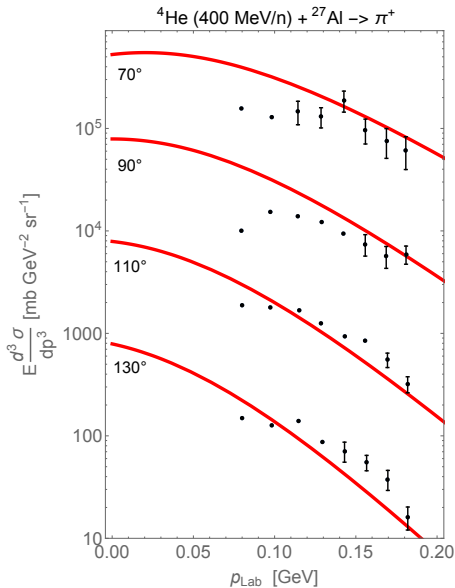
# PION MODEL RESULTS - HIGH ENERGY PROTON PROJECTILES



Cross section for 8.8° multiplied by 10

- DDFRG model agrees well with data

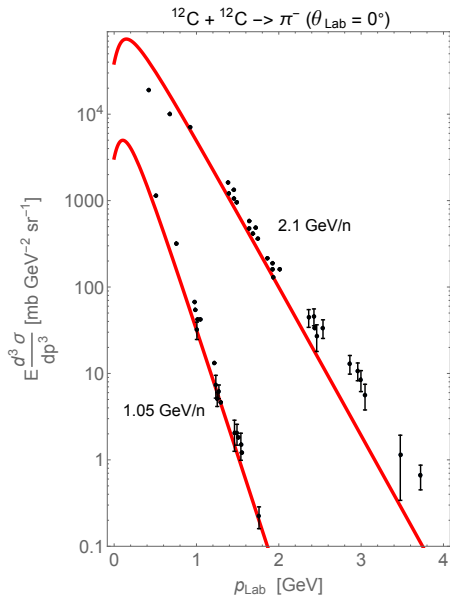
# PION MODEL RESULTS - LOW ENERGY $^4\text{He}$ PROJECTILES



Cross sections multiplied by increasing factors of 10

- DDFRG model agrees well with data, but some differences

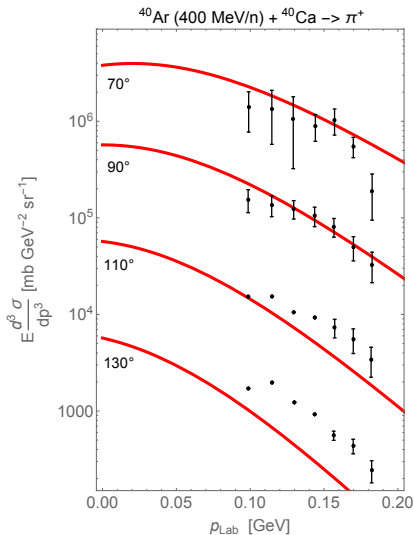
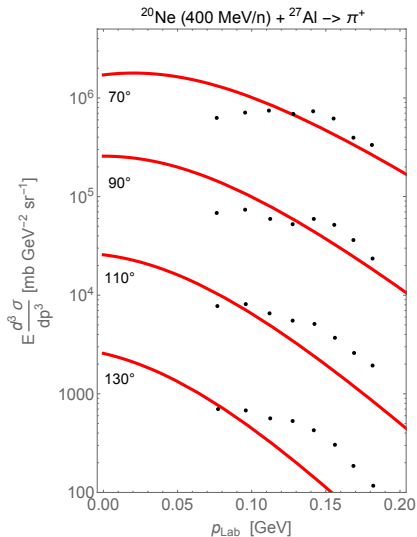
# PION MODEL RESULTS - LOW ENERGY $^{12}\text{C}$ PROJECTILES



Cross sections for 2.1 GeV multiplied by 10

- DDFRG model agrees well with data, but some differences

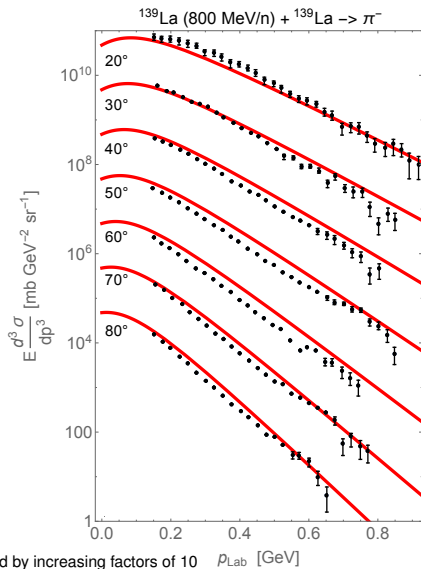
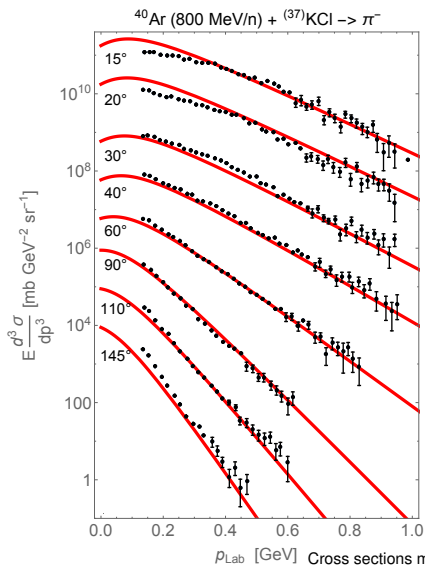
# PION MODEL RESULTS - LOW ENERGY $^{20}\text{Ne}$ AND $^{40}\text{Ar}$ PROJECTILES



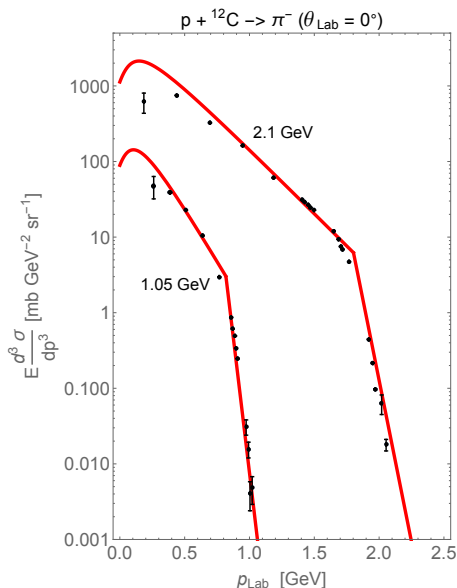
Cross sections multiplied by increasing factors of 10

- DDFRG model agrees well with data, but some differences

# PION MODEL RESULTS - LOW ENERGY $^{40}\text{Ar}$ AND $^{139}\text{La}$ PROJECTILES



# PION MODEL RESULTS - LOW ENERGY PROTON PROJECTILES



Cross sections for 2.1 GeV multiplied by 10

- DDFRG model agrees well with data, but some differences at low pion momentum
- Relativistic kinematic calculation of maximum energy cut-off agrees perfectly with data!
- Hypothesis: Data beyond cut-off due to Fermi motion of nucleons
- This reaction is unique!
  - Projectile is simple (proton)
  - Projectile energy not very large
  - Pions negatively charged (spallation)

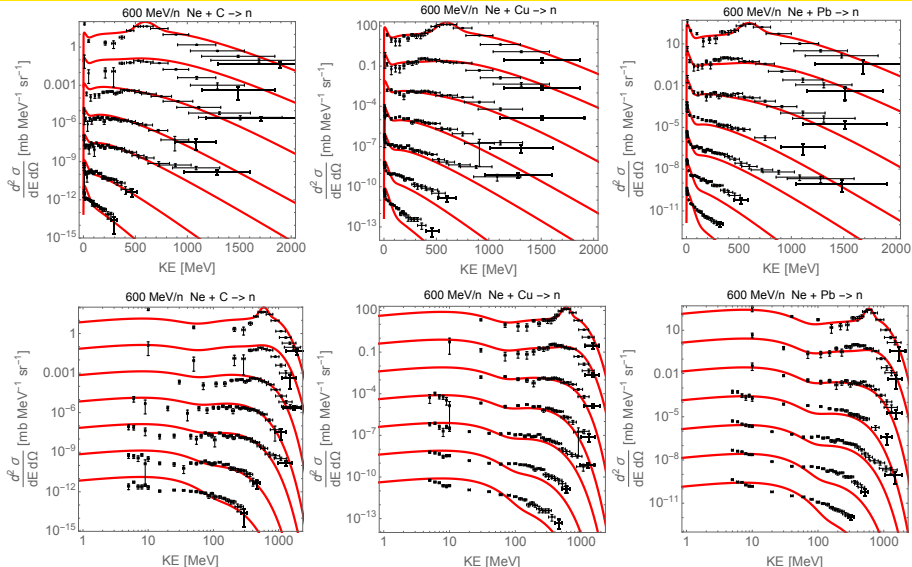
## 6. NEUTRON MODEL (DDFRG3)

- Just adapt Proton model?
  - all four sources
- **Complication # 1: MASSIVE amount more data**
  - A good thing, but very time consuming to analyse
  - Nakamura, Heilbronn, *Handbook on secondary particle production and transport by high-energy heavy ions* (World Scientific, Singapore, 2006)
  - BEVALAC (337 MeV/n), HIMAC (230 - 600 MeV/n), RIKEN (95, 135 MeV/n)
- **Complication # 2: BEVALAC and HIMAC data disagree!**
- **Complication # 3: No data above 1 GeV/n**
- **Complication # 4: Neutron spectra display prominent low energy peak not seen in proton spectra**

- Simple modification (parameters) of DDFRG proton model fails
  - Unable to agree with experiment at very low energy
  - Unable to describe low energy target fragmentation neutron peak
    - even with severe tweaking of target contribution parameters
- New Method:
  - Continue using PROJECTILE fragmentation code, NUCFRG, as normalization for projectile fragmentation, projectile direct knockout and central fireball (but not target contributions)
  - Use NUCFRG in inverse kinematics, i.e. swap projectile and target, but same kinetic energy / nucleon
  - NUCFRG in inverse kinematics = target fragmentation
  - Normalize target fragmentation spectra to NUCFRG inverse kinematics
  - New method much more successful



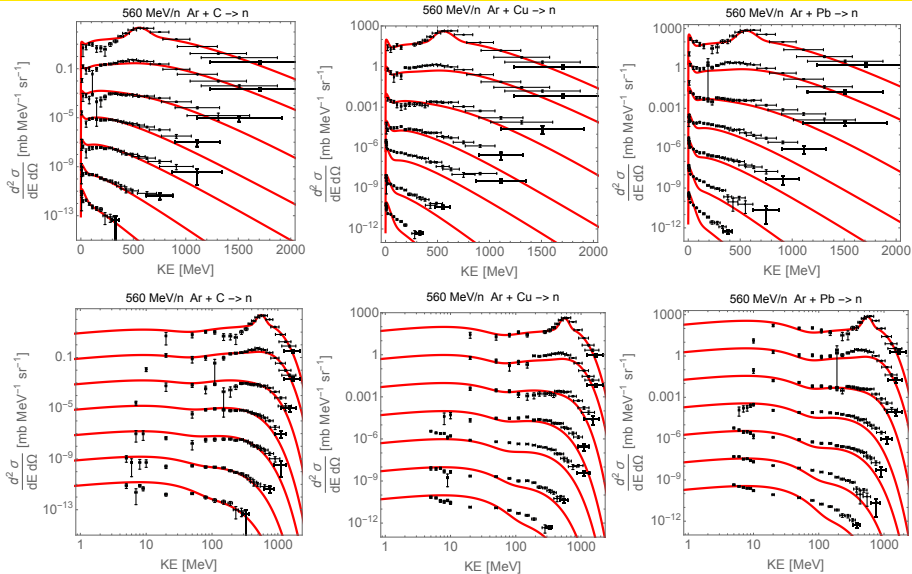
# NEUTRON MODEL RESULTS - DDFRG VS. HIMAC DATA



Log-Linear (top row) and Log-Log (bottom row) [Data from Nakamura and Heilbronn]

Angles top to bottom:  $5^\circ$ ,  $10^\circ$ ,  $20^\circ$ ,  $30^\circ$ ,  $40^\circ$ ,  $60^\circ$ ,  $80^\circ$ . Cross sections multiplied by increasing factors of 0.1

# NEUTRON MODEL RESULTS - DDFRG VS. HIMAC DATA

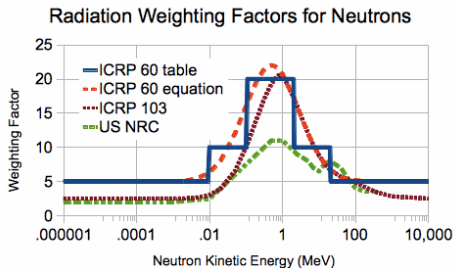


Log-Linear (top row) and Log-Log (bottom row) [Data from Nakamura and Heilbronn]

Angles top to bottom: 5°, 10°, 20°, 30°, 40°, 60°, 80°. Cross sections multiplied by increasing factors of 0.1

# LOW ENERGY NEUTRON PEAK

- Low energy neutron peak  
- No Coulomb barrier



[Wikimedia commons]

- Not present in proton data  
- Coulomb barrier present

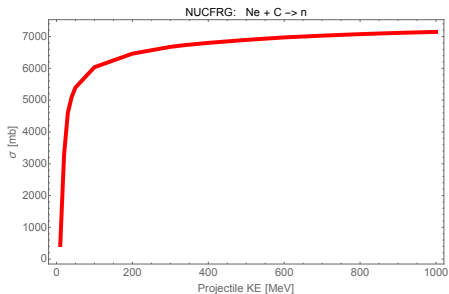
# DATA DISAGREEMENTS

- Foundational problem in space radiation
  - Neutrons most important secondary particle in space radiation
- $\text{Ne} + \text{C}, \text{Cu} \rightarrow \text{n}$ 
  - BEVALAC 337 MeV/n
  - HIMAC 400 MeV/n
- $\text{He} + \text{Al}, \text{Cu} \rightarrow \text{n}$ 
  - RIKEN 135 MeV/n
  - HIMAC 230 MeV/n
- Energies not the same, but close enough for comparison
  - NUCFRG calculation and DDFRG calculation will show this

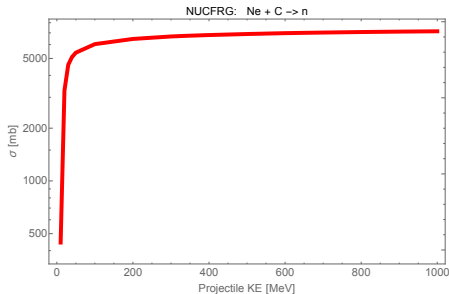
# BEVALAC vs. HIMAC DATA DISAGREEMENT

- Ne + C NUCFRG expectations:

- BEVALAC, 337 MeV/n:  $\sigma = 6729$  mb,  $\text{Log}_{10}6729 = 3.828 = 3.83$
- HIMAC, 400 MeV/n:  $\sigma = 6801$  mb,  $\text{Log}_{10}6801 = 3.833 = 3.83$
- Spectra plotted on Log scale should lie on top of each other, especially low neutron energy

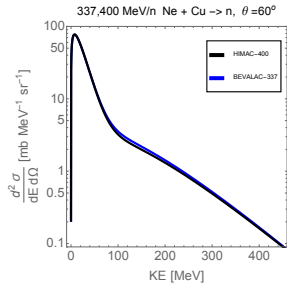
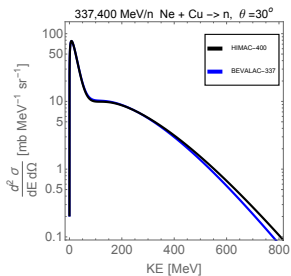
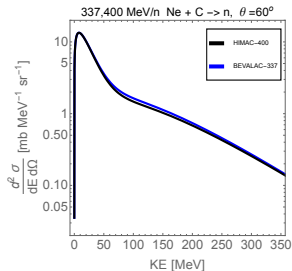
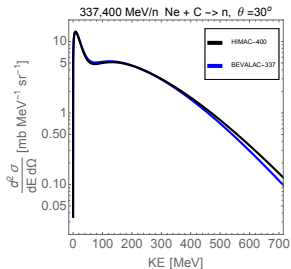


Linear



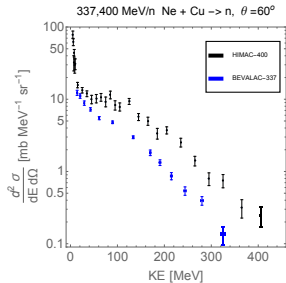
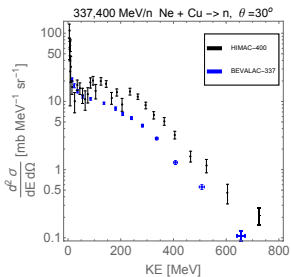
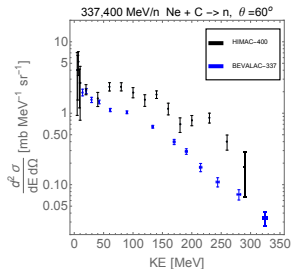
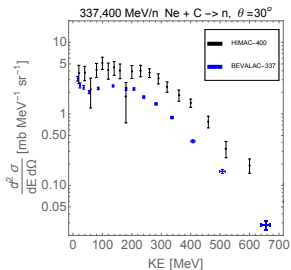
Log

# BEVALAC vs. HIMAC DATA DISAGREEMENT



DDFRG predicts no difference between 337 MeV/n (BEVALAC) and 400 MeV/n (HIMAC) data

# BEVALAC vs. HIMAC DATA DISAGREEMENT



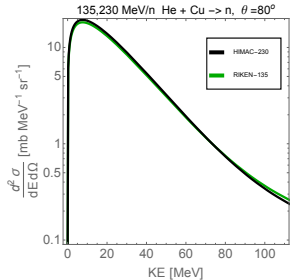
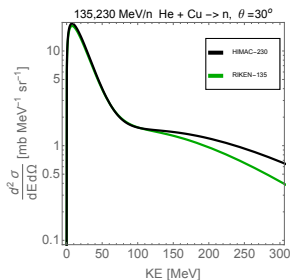
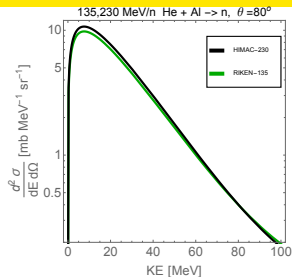
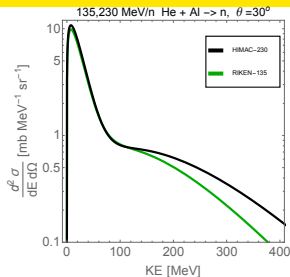
BEVALAC and HIMAC data disagree

# RIKEN vs. HIMAC DATA DISAGREEMENT

- He + Al NUCFRG expectations:
  - RIKEN, 135 MeV/n:  $\sigma = 719$  mb,  $\text{Log}_{10}719 = 2.857 = 2.9$
  - HIMAC, 230 MeV/n:  $\sigma = 841$  mb,  $\text{Log}_{10}841 = 2.925 = 2.9$
  - Spectra plotted on Log scale should lie on top of each other, especially low neutron energy

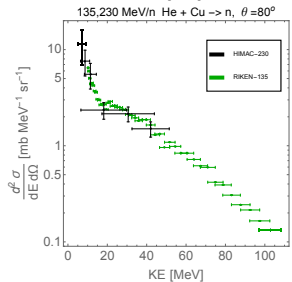
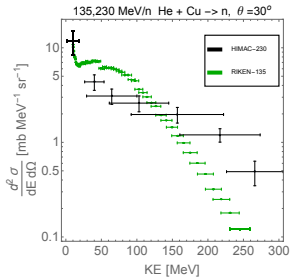
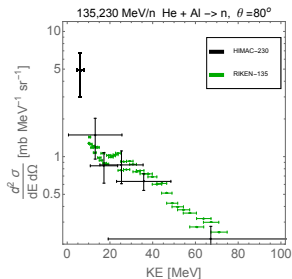
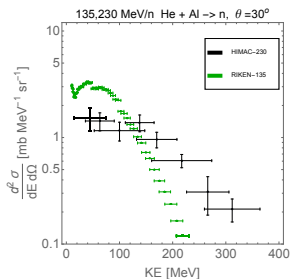


# RIKEN vs. HIMAC DATA DISAGREEMENT



DDFRG predicts no difference between 135 MeV/n (RIKEN) and 230 MeV/n (HIMAC) data  
below 100 MeV

# RIKEN vs. HIMAC DATA DISAGREEMENT



Data disagree for  $30^\circ$  (below 100 MeV)

Data agree for  $80^\circ$

## 7. SUMMARY, CONCLUSIONS AND FUTURE WORK

- **Proton and Neutron production**
  - Three thermal sources - Projectile, Target, Central fireball
  - One direct source - accounts for quasielastic peak at beam rapidity
- **Light ion production**
  - Three thermal and one direct source - Projectile, Target, Central fireball
  - Modified coalescence model based on proton model
- **Pion production**
  - One thermal source - Central fireball
- **DDFRG overall very good agreement with experimental data**
  - Successes emphasized - complete comparisons in publications
- **Future work**
  - Resolve data discrepancies
  - Expand neutron data comparisons
  - Pion projectiles
  - Compare to world nuclear physics models
  - Verification and Validation Transport
  - Neutron data availability: Nothing above 1 GeV/n

## 7. SUMMARY, CONCLUSIONS AND FUTURE WORK

- Neutron space radiation - Are foundations solid ???
  - Low energy neutron fragment cross sections not measured  
(Biological weighting factor maximum at low neutron energy)
  - No cross section data above projectile kinetic energy of 1 GeV/n
  - Cross section measurements disagree
  - Transport codes disagree (e.g. MSLRAD)
- Neutron detectors on Mars, ISS and Artemis very important

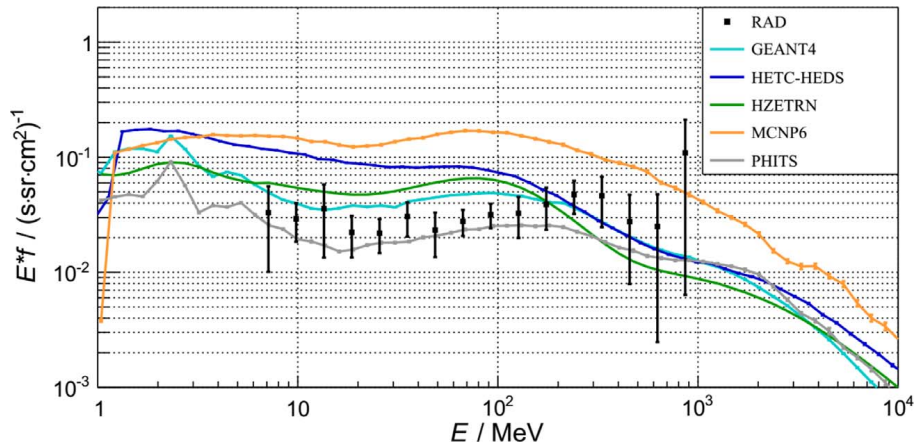
# ACKNOWLEDGEMENTS

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**THE END**

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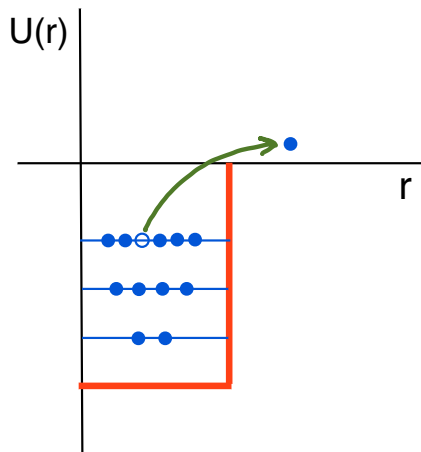
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Matthia et al., Life Sci. Space Res. 14, 18, 2017

*Significant discrepancies*

# COULOMB BARRIER

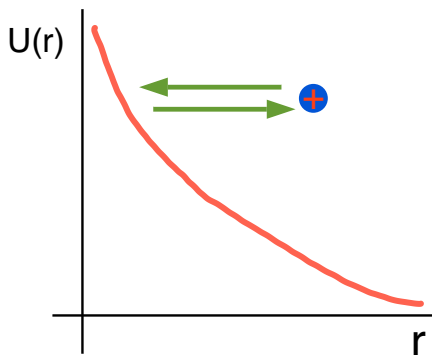


Neutrons in attractive nuclear potential easily knocked out

Low energy target fragmentation peak in neutron spectra

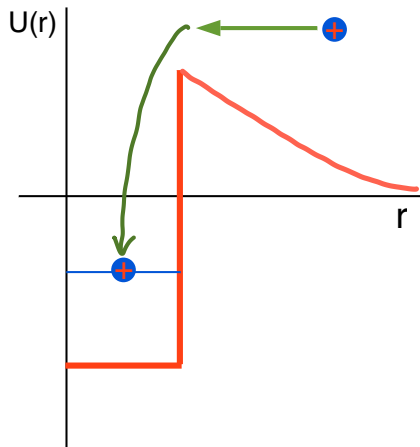


# COULOMB BARRIER



External charged particles repelled by pure Coulomb potential

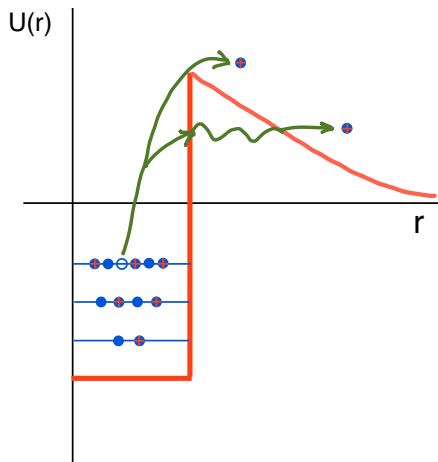
# COULOMB BARRIER



Nuclear + Coulomb potential

→ External charged particles overcome Coulomb barrier

# COULOMB BARRIER



Internal protons not easily knocked out, due to Coulomb barrier

Absence of low energy target fragmentation peak in proton spectra